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A.B.Ospanov, A.N.Vasilyev, D.A.Budnikov,
D.K.Karmanov, A.A.Vasilyev, D.Sh.Baymuratov,
B.O.Toxanbayeva, D.B.Shalginbayev

IMPROVEMENT OF GRAIN DRYING AND DISINFECTION PROCESS IN THE MICROWAVE FIELD

Monography

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The monography includes the results of the theoretical and experimental study on the heat treatment process of grain in an electromagnetic microwave field (EMMF). Mathematical and computer models of grain heating in the microwave field, heat and moisture exchange in the grain layer under microwave convection influence of the microwave electromagnetic field in the working zone are presented. Experimental data on the determination of the parameters and operating modes of the microwave module and automated control of the drying process and disinfection of grain are also presented in this study.

The monography can be useful for specialists of grain processing enterprises, scientific organizations, design bureaus, undergraduates and doctoral students engaged in the operation, calculation, design and scientific research of grain dryers.

Reviewers:
R. A. Omarov - Doctor of Technical Science, Head of the Energy Saving, Automation and Renewable Energy Laboratory at the "Kazakh RI of Mechanization and Electrification of Agriculture" LLP,
A.I. Iztaev – Doctor of Technical Science, Academician of the National Academy of Sciences of Kazakhstan, Director of the Institute of Food Technology of Almaty Technological University

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INTRODUCTION

In the State Program for the Development of the Agro-Industrial Complex of the Republic of Kazakhstan for 2017-2021 (No.420 dated February 14,2017) declares that Kazakhstan has good prospects for further development of the following sectors: the export positions of the oilseed and meat; in terms of grain and flour, Kazakhstan has become one of the largest exporting countries in the world. The membership of Kazakhstan and Russia in the Eurasian Economic Union creates opportunities and simultaneously high demands on competitiveness in both the local and foreign markets.

Increasing production and improving the quality of grain is the most important objective, the solution of which ensures the food security of EEU countries. At the same time, this solves not only the objectives of producing the required volume and quality of bakery products and raw materials for the food industry, but also obtaining concentrated feed for livestock and poultry. The deep processing of grain is of particular relevance today. Therefore, the elimination of grain losses, the preservation of its commercial qualities, is an urgent issue.

However, it is generally known that in the post-harvest treatment and storage process, about 10% of the gross grain harvest is lost. This happens due to the untimely drying of the grain, pests and microorganisms damages. Depending on the technology of tillage, up to 30% of the grain can be infected with mycotoxins after harvesting. This grain is impossible to use in bakery or feed production without special treatment . Therefore, drying and disinfection are important elements of the technological chain for grain treatment and storage.

The energy intensity of the drying process is not less than 20% of energy consumption for grain production. Therefore, reducing the energy intensity of the process and increasing productivity is one of the main directions in scientific studies on this topic. In this case, the use of electrotechnologies makes it possible to approach a complex solution to the problem of grain drying and disinfection. The work considers the possibilities of using a microwave field to intensify the drying and disinfection of grain.

The authors analyzed current trends in the use of the microwave field units for grain and feed drying or disinfecting. An attempt was made to theoretically justify the necessary modes of grain treatment. The obtained mathematical and computer models were used to demonstrate the effect of microwave constructive features on the efficiency of grain drying and disinfection.

CHAPTER 1 TO THE CLASSIFICATION OF GRAIN DRYING EQUIPMENT

Various types of units are used for grain drying, which can be classified according to various characteristics. We will give a possible classification of grain drying constructions depending on the aggregate state of material and grain layer characteristics, taking the classification used by A.S.Ginzburg as a basis. [1]:

A) *DRYING IN DENSE LAYER*:

a) Drying in dense fixed layer:

- hopper;

- floor;

- in silos of elevators;

b) Drying in dense moving layer: conveyor (Fig.1.1); belt; column (fig. 1.2); louvered (fig. 1.3); hopper (fig. 1.4, 1.5); tower (fig. 1.6).

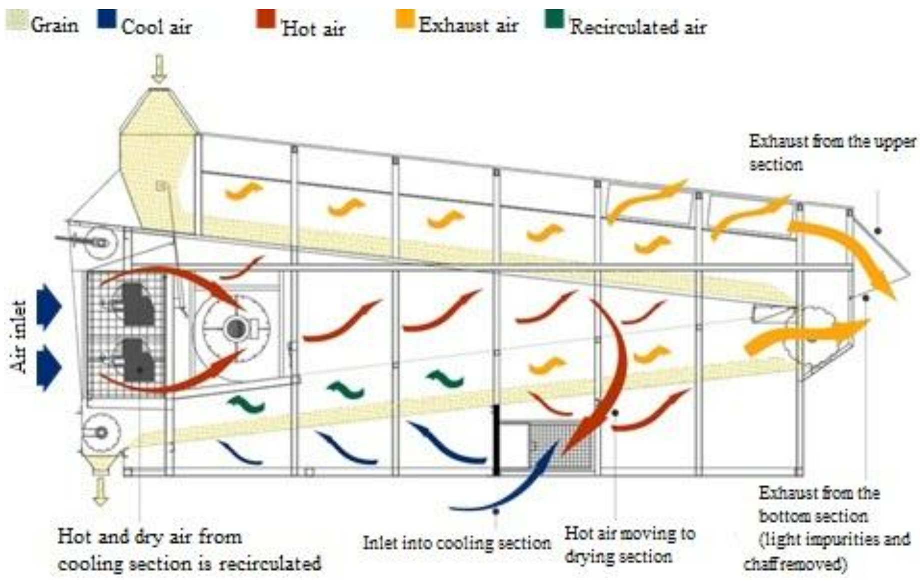
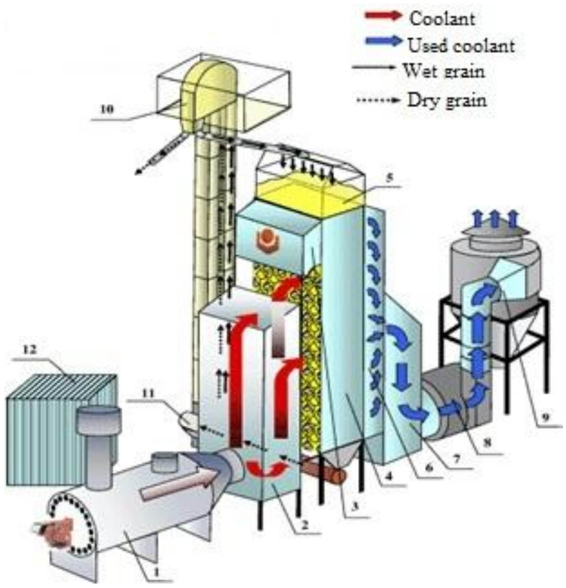


Figure 1.1 - Technological process in the conveyor grain dryer [2, 3]

The advantage of any conveyor dryer is the ability to dry grain in a thin layer. This makes it possible to ensure the uniformity of heating up and moisture removal over the entire volume of grain on the conveyor. The peculiarities of this dryer type include material used for conveyors, air distribution system of both hot coolant and the air used for cooling the grain. There are stationary and mobile constructions. Diesel fuel or gas can be used as a coolant.

This type of grain dryer is manufactured for high productivity (up to 50 tons per hour) of grain drying, which is in-demand when harvesting maize and late varieties of sunflower. However, the proposed modern constructions of grain dryers are characterized by increased energy intensity due to inefficient use of the coolant energy, imperfect air distribution system and almost complete absence of recuperation.

Column grain dryers (Figure 1.2) are historically most known to our consumer. These dryers got their name because of the drying chamber construction, which is made in the form of a vertical column where grain moves by gravity. The drying



1- heat generator; 2 - supply channel; 3 - channel of coolant supply; 4 - column; 5 - over-the-drier hopper; 6 - coolant return channel; 7 - connecting channel; 8 - fan; 9 - aspiration system; 10 - double-flow cup elevator; 11 - discharge screw conveyor; 12 - operator's workplace

Figure 1.2 - Scheme of the column grain dryer [6]

agent is fed into the drying chamber through ducts, that go across the drying chamber. Then the used coolant is cleaned. Further progress of coolant depends on whether the elements of recuperation are used in a grain dryer. On the column dryers the technologies of intensification and reduction of energy intensity for drying processes of grain are thoroughly studied through the use of recycling and preheating [4,5,6,7]. We will not dwell on these technologies because our work is focused on the other.

In the construction of many grain dryers the main objective is to ensure uniform distribution of the coolant in the grain layer. These objectives can be solved by the louvered grain dryer (Figure 1.3).

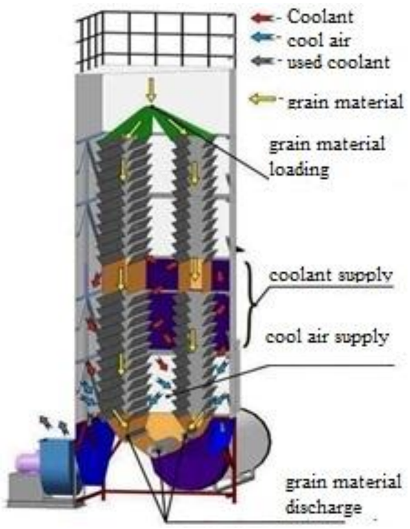


Figure 1.3 - Drying scheme in the louvered (column) grain dryer [8]

Today mobile hopper grain dryers are becoming popular (Figure 1.4). The construction of the drying chamber provides a variable thickness of the grain layer along the hopper height. In the upper layers it is larger, in the lower layers it is smaller, which makes it possible to change the intensity of heat and moisture exchange.

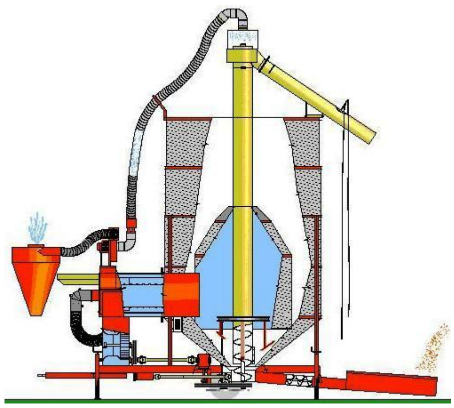


Figure 1.4 - Construction of hopper grain dryer [9, 10]

Mixing of grain during the drying process can be ensured by moving it from the upper zone of the hopper to the bottom with a vertical auger.

In tower grain dryers (Figure 1.5) drying of the grain layer annulus is provided. The construction provides a slight uneven drying over the grain layer.

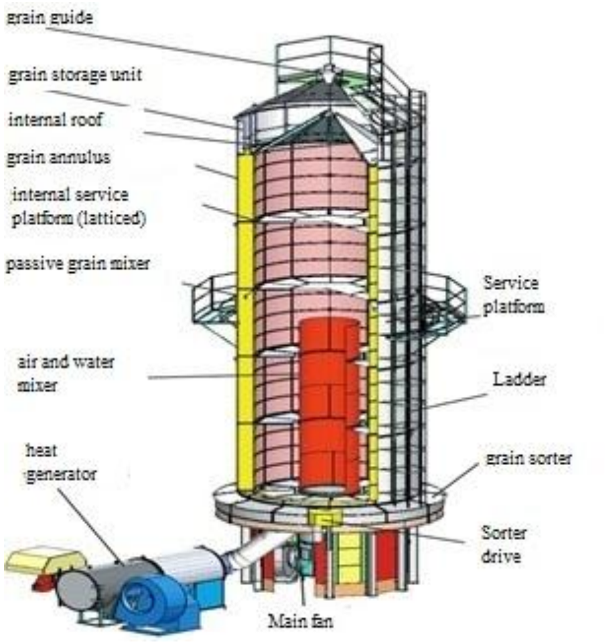


Figure 1.5 - Scheme of the tower grain dryer [11]

B) *DRYING IN LOOSENED LAYER*:

- Drum (fig.1.6); auger (fig.1.7); cascade; circular (fig.1.8); vibrational (fig.1.9).

Forage grains are usually dried in drum grain dryers, because it is not always possible to observe the required temperature regime. Heat exchange is carried out by the coolant and heated walls of the rotating drum. The drum is installed at an angle, which ensures the movement of grain along to the discharge zone.

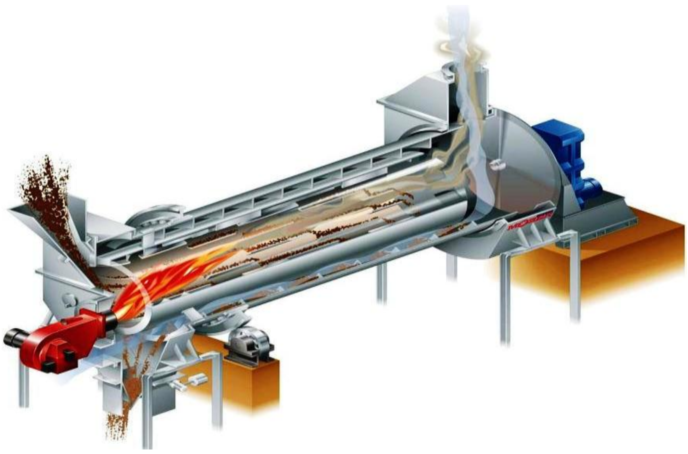


Figure 1.6 - Drying scheme in the drum dryer [12]

The grain in auger dryers is transported by means of an auger. During the movement the grain is blown over by heated air.

Figure 1.7 - General view of an auger dryer [13]

In the circular grain dryer (Figure 1.8), a layer of grain is formed on the rotating perforated surface through which hot air is blown. Rotation of the grain layer provides a more even distribution of the drying agent and more uniform drying of the grain layer.

Figure 1.8 - General view of circular grain dryer [14]

Vibrational dryers are used to ensure a more uniform flow of drying material by the drying agent during vibration (Figure 1.9). As a rule, these dryers are not used for grain drying.

Figure 1.9 - General view of a vibrational dryer [13]

B) DRYING IN THE PSEUDOLIQUEFIED (BOILING) LAYER:

- single chamber; multichamber; spouting layer (fig.1.10); with vibroaeroliquefied layer; with vibroaerospouting layer.

A special advantage in the uniformity of heat and moisture exchange is held by the grain dryers with drying in pseudoliquefied (boiling) layer. Fig. 1.10 shows drying scheme in the spouting layer. It should be borne in mind that this technology does not provide high drying performance. Also, frequent hits of grain during the drying, makes it unreasonable to use these units for seed grain.

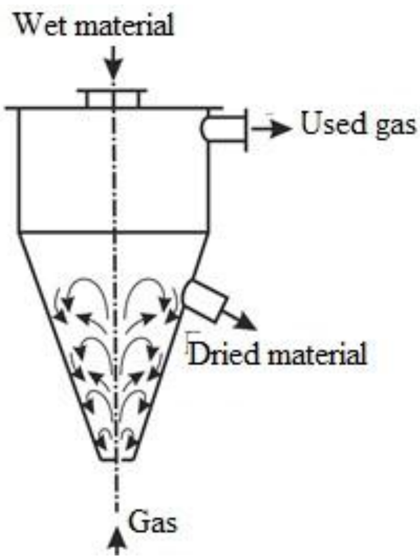


Figure 1.10 - Drying scheme in the spouting layer

Each type of presented grain dryers has its own advantage, its niche in a wide range of technological processes. As a rule, they have been developed for the conditions of specific technological processes.

The issues of grain drying are sufficiently described in the works of A.V.Avdeev, A.P.Gerzhoi, A.S.Ginzburg, A.V.Golubkovich, A.V.Likov, N.I.Malina, G.S.Okun, S.D.Pticyn, V.A.Rezhikova, S.P.Rudobashty, V.F.Sorochinskiy, A.G.Chizhikov. In more detail, the drying units in a dense fixed layer of grain, the improvement of structures and management of the process were studied by V.I.Aniskin, T.N.Bastron, I.F.Borodin, A.N.Vasiliev, N.L.Girnyk, G.A.Gulyaev, V.R.Krausp, S.K.Manasyan, B.E.Melnik, O.V.Pilyaeva, N.V.Cuglenok. These scientists have carried out a huge amount of work in this direction, which allowed the authors to try to delve into some aspects of heat and moisture exchange, and improve drying technologies using electrotechnologies. The presented material emphasizes the use of microwave fields.

CHAPTER 2 USING MICROWAVE FIELDS FOR INCREASING GRAIN DRY EFFICIENCY

2.1. The principles underlying the improvement of the grain drying technology

The main peculiarity of technological processes in agriculture is work with biological objects. This peculiarity despite of creating great difficulties, both in organization of production, and in management of equipment and technological processes, has certain advantages. It allows you to use energy of the biological object itself to solve the problems of product manufacturer. The objective is to release this energy stored by biological object, and learn how to manage it.

The solution of this problem has been considered by the authors using the example of technologies for post-harvest and pre-seeding grain treatment [15, 16]. Several approaches (adaptive, energetic, thermodynamic, informational) have been considered, allowing to describe response of a biological object to the external technological influences. As a result of the analysis of each approach, the object's reactions to external influences or requirements to the technological process have been formulated, the implementation of which shall allow managing the biological reactions of the object for solving production problems.

The following have been established using the *Adaptive approach* [17]:- for the passing of adaptation reactions at the level of non-structural, but functional changes, the impact should be gradually increasing, prolonged, intermittent (cyclic);

- to obtain an activation reaction, a physical effect of a certain value is necessary, then it should be reduced or stopped. Then increase the force of impact by 20-25% relative to the original value;

- the initial value of the impact should be minimal for the passing of adaptation reactions on the lower levels of reactivity. At the same time, the level of reactivity will be the highest;

- the greatest effect from physical impact will be received combining non-specific reactions with specific ones.

The Energetic approach [18] lead to the following conclusions:

- the principle of energy saving is not always determinative in the behavior and development of biological objects and cannot be accepted as the main one;

- the self-regulation system of the kernel biological processes has several stationary states, determined by varietal features, "experience" of previous developments and post-harvest treatments.

The thermodynamic approach [19,20] made it possible to formulate the principles of grain reactions to external influences:

- the level of external influence on the grain can stop the transfer of grain into another stationary state. In this case, adaptation reaction will be in the "training" stage;

- with external influence, when the grain is transferred into another stationary state, the efficiency of the pre-seeding treatment and duration of the effect depends on the value of entropy corresponding to the new stationary state;

- the time during which grain will remain in the new stationary state will be different depending on the following aspects: a) how grain was taken away from the initial stationary state (what kind of external influence has been applied); b) what value of entropy corresponds to the new grain state.

The informational approach [21 - 24] allowed us to formulate the *main principles* [25,26], which must be observed when developing methods for *pre-seeding treatment and drying of grain* [27,28]:

- it is necessary to determine the minimum energy of external influence, which is taken by grain;

- the influence must be cyclic. The cycle periods depend on the seed and type of biophysical effect;

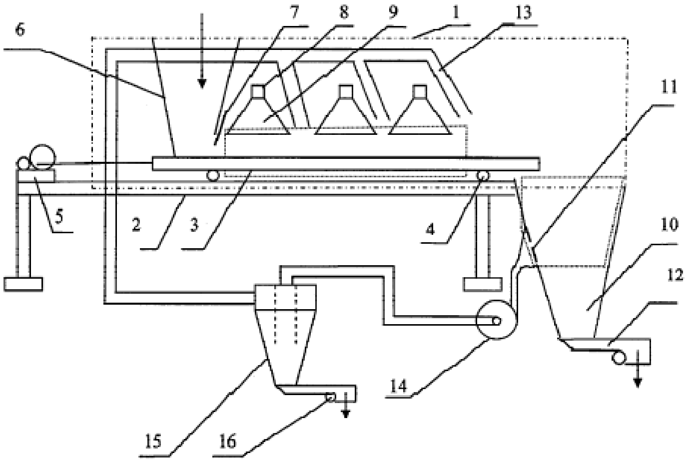
- the amplitude of external influence in each cycle can be different (increasing from cycle to cycle);

- to improve the efficiency of drying, it is advisable to change the types of external influences in the process of heat and moisture exchange.

The greatest effect in this case can be obtained by using electrotechnologies because their energy intensity is much lower than energy intensity of thermal effect. Let us consider some variants of using microwave fields for increasing the efficiency of grain drying.

2.2 Variants of microwave fields usage for increasing the efficiency of grain drying

A sufficiently active use of microwave energy for drying, including grain, have been developing in the 80's of the last century. In the Soviet Union, this direction has been intensively developed in the Institute for Strategic Research named after V.P. Goryachkin under the leadership of Academician I.F. Borodin [29,30,31]. The issues of soil disinfection and pre-seeding seed treatment have been developed here. The questions of using microwaves for grain drying were considered by V.I. Pakhomov. [32]. For the most part, such developments have been conducted abroad. In 1982, Ken Bratney Co began producing microwave grain dryers [33]. In this unit the multichamber drying section is located horizontally. The grain is transported by an auger conveyor. Magnetrons with a power of 1.3 kW each, are installed on the drying vacuum chambers above the conveyor. 9 magnetrons are used for one drying chamber. The manufactured construction had 4 chambers. Combination of microwave heating with vacuum provided energy intensity of drying 0.13 ... 0.26 kWh per 1 kg of evaporated moisture. The cost of such grain dryer was about 100 thousand dollars. The "conveyor" concept in microwave grain dryers has found its reflection in the works of Soviet scientists. Application of microwave field for grain treatment moving on the conveyor has been used by V.I. Pakhomov in his work. [32]. This trend has also been reflected in modern works on microwave drying [34]. Figure 2.1 shows one of the options for implementing such unit.



1- body of the working chamber; 2- support frame; 3 - device for material moving; 4 – support roller; 5 – drive; 6 – charging device; 7 - adjustable slide gate; 8 - microwave radiators; 9 – sockets; 10 - discharge device; 11 – slotted screen; 12 –bucket-wheel feeder; 13 - devices for supply and bleed air in the form of a duct system; 14 – dust-arresting device; 15 –bucket-wheel feeder; 16 – ducting; 17 - fan

Figure 2.1 Scheme of installation for drying bulk dielectric materials under the patent [34]

Under the influence of microwave field the material heats up, the moisture contained in it, evaporates. When the dried material is discharged, it enters discharge device 10 on slotted screen 11, into the space under which the air is pumped by fan 17 through the holes in screen 11 to discharge device 10, creating a blowing zone inside it, using hot air, pumped from the heating zone by duct system 13, cleaned by dust-arresting device 14 and fed through duct 16 to fan 17.

The conveyor dryers are more popular today. The series of microwave conveyor dryers MDBT (manufacturer Linn High Therm , Germany) is designed for drying small items (Figure 2.2). The block structure of the conveyor allows you to vary its length from 5 to 30 m. Conveyor width varies from 200 to 1000 mm.

Figure 2.2 - Conveyor dryer Linn MDBT [35]

The maximum temperature for heating the product is 230°C. The unit can be used to heat or remove water from food products, as well as in all processes where it is necessary to remove water or preheat any microwave-absorbing material. Fig. 2.3 shows drying of carrots on Linn MDBT unit.

Figure 2.3 - Drying carrots on Linn MDBT unit

A microwave conveyor dryer has been developed and manufactured by AST company for bread drying (Figure 2.4). Modification of this unit allows to use it for other bread products and forage treatment.

Figure 2.4 - Microwave dryer AST-4 [36]

Chinese company Liaoyang Korican Machinery Co., Ltd. offers a conveyor microwave unit (Figure 2.5) for drying medicinal materials at low temperatures and heat sensitive materials in such areas as food industry, bioengineering, chemistry . A few technical characteristics of the unit are given in Table 2.1

Figure 2.5 - Conveyor microwave unit [37]

Table 2.1 - Technical characteristics of microwave conveyor dryer

Model	**KWZD-20**	**KWZD-30**	**KWZD-50**	**KWZD-100**	**KWZD-150**
Operating frequency of microwaves, GHz	2450±15				
Total power of supply, kW	<30	<45	<70	<125	<180

Microwave conveyor unit produced by "Ingredient" LLC Barkhan-3 (Figure 2.6) has the following technical characteristics:

Number of microwave modules: 2pcs.

Nut productivity is 400 kg / h .

Sunflower seeds productivity is 250-300 kg / h.

Operating temperature: from +5 to +230 ^{o}C.

Power supply: 3f 380V, 50-60 Hz.

Power consumption, max: 60 kW.

Overall dimensions (LxWxH): 3600x1300x2300 mm.

Weight: 850 kg.

This unit can be used for: roasting nuts, pistachios, peanuts, almonds and others, as well as seeds and sunflower kernels; drying of dry mixtures, cereals, fodder for livestock and others; drying of natural and industrial bulk materials, such as clay, sand, carbon, construction and chemical materials; "Dry cooking" of vegetables, fruits and other products.

In this case, the product is cooked in its own evaporated liquid, the final product quality in this case is much higher than in traditional cooking in water, since the useful substances do not go into the water, but remain in the product; seed treatment before seeding, while the seed germination rates increases and the drying efficiency is much greater than when frying; termination of small bugs that reproduce in products, and restoration of the product quality.The Chinese company Shandong Adasen Trade Co., Ltd offers a microwave conveyor unit JN-15 for drying and pasteurizing products (Fig. 2.7).

Figure 2.6 - Barkhan-3 unit [38]

Figure 2.7 - Microwave unit JN-15 [39]

Brief technical characteristics of the unit are given in Table 2.2

Table 2.2 - Technical parameters of JN-15

Power-supply voltage	380 V, 50 Hz
Output frequency	2450 + 50 MHz
Full power	21kva
Microwave field power	0-15kW (adjustable)
Heating temperature	0-200 ^{o}C (adjustable)
Dimensions (LxWxH)	8300x850x2050 (mm)

As can be seen from the presented data, the productivity of conveyor units is not high. They are mainly designed for drying small batches of vegetables, spices, nuts, seeds, etc. It is difficult to imagine a high-performance (20 ... 50 t / h) conveyor system in the grain drying line. Achieving such productivity is possible by increasing the length of the conveyor, which is very problematic.

The increase in the efficiency of microwave dryers is carried out, as in the units produced by Ken Bratney Co, due to the use of suction in vacuum drying chambers. Thus, in the microwave units of Musson series (Figure 2.8), the drying of products takes place in a vacuum chamber at a lower temperature than at atmospheric pressure, as a rule, this is enhanced drying at a temperature of 30 ° C.

Figure 2.8 - Microwave vacuum unit "Musson" [40]

Nevertheless the temperature of microwave drying in a vacuum is low, the liquid in the product has a temperature close to the boiling point. This method of drying ensures maximum preservation of vitamins and nutrients in the final product. This is especially important when drying pharmaceuticals, medicinal plants and herbs etc. [40].

The unit is designed for low-temperature vacuum drying of products such as: biologically active additives, medicinal herbs and roots, pharmaceutical materials, seafood; sterilization of phyto-tea, food additives, spices, etc .; modification of the product structure: foamed skins etc; distillation of plant material to produce dry residue and flavored distillate; nuts frying, sunflower seeds; study of the combined effects of microwave energy and reduced pressure.

One of the variants of microwave vacuum dryers has a drum structure. This device is presented in the patent [41].

The microwave vacuum unit "Alternative" [42] (figure 2.9) is recommended for dehydration of expensive products: meat, cheese (70-80 kg/day), gelee royale, extracts, candied fruits, etc., for roasting coffee, nuts. the payback period of MU for these products is 3-12 months. Vegetables are dried perfectly in these plants. The final products of evaporation can be obtained with different moisture level, for example: balyks with moisture content of 28-55%, candied fruits 28-37%, onion 12-14%, and gelee royale 1.2-2.0%, etc.

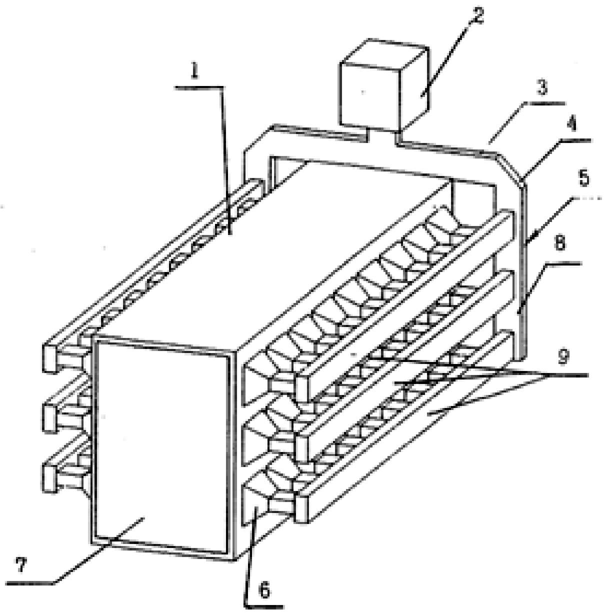
Figure 2.9 - Microwave vacuum unit "Alternative" [42]

The use of vacuum significantly increases the efficiency of microwave dryers of wood (Figure 2.10). Microwave vacuum technology is an effective technology for drying massive wood of large cross-section; with this technology it is possible to dry timber, log, half timber up to 10% of humidity.

Figure 2.10 - Microwave vacuum dryer of wood [43]

This method of drying makes it possible to produce walling material, which, according to its consumer properties and quality is very close to the laminated beam, and even surpassed it in some respects.

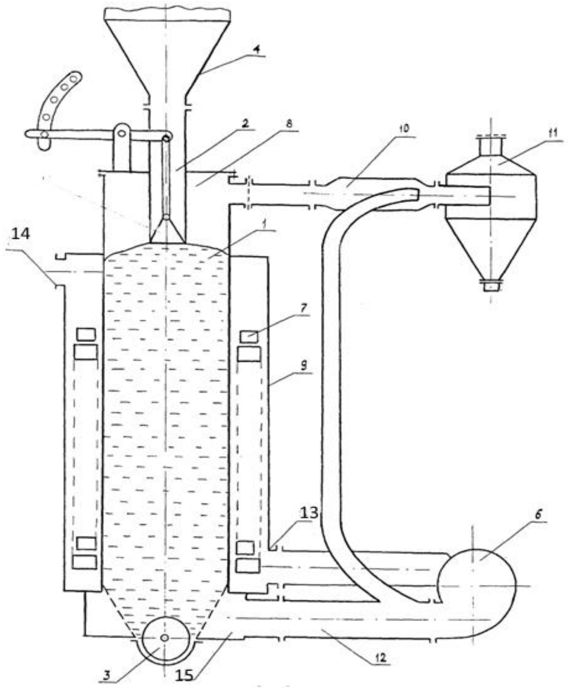
Distribution of the wood over the large volume of drying chamber requires a uniform distribution of the microwave field. One of the solutions to this problem is reflected in the patent (Figure 2.11) [44].



1- drying chamber; 2 - microwave generator; 3 - T-junction waveguide; 4 - waveguide bends; 5 - wave-guide ducts; 6 - horn antennas; 7 - charging hole; 8, 9 - vertical and horizontal waveguides.

Figure 2.11 - Unit scheme for drying dielectric materials according to the patent [44]

Grain dryers used in post-harvest processes must be organically integrated into production lines and provide the required productivity. The microwave dryers, described earlier provide good quality of the material after drying; these dryers have low energy intensity, but have low productivity. In this regard, flow-type grain dryers are more effective; in these dryers grain under its own weight moves from top to bottom. During the movement, it is exposed to the microwave field and drying agent. Fig. 2.12 shows one of the grain dryer schemes (Figure 2.12) [45].



1- drying chamber; 2 – charging device; 3-discharge device in the form of an auger; 4, 5 - charge and discharge hoppers; 6 - fan; 7 - magnetrons with power supplies; 8- suction housing; 9 - casing under the microwave generators; 10 - injector; 11 - cyclone; 12 - junction; 13,14 - inlet and outlet adapters; 16 -draft chamber

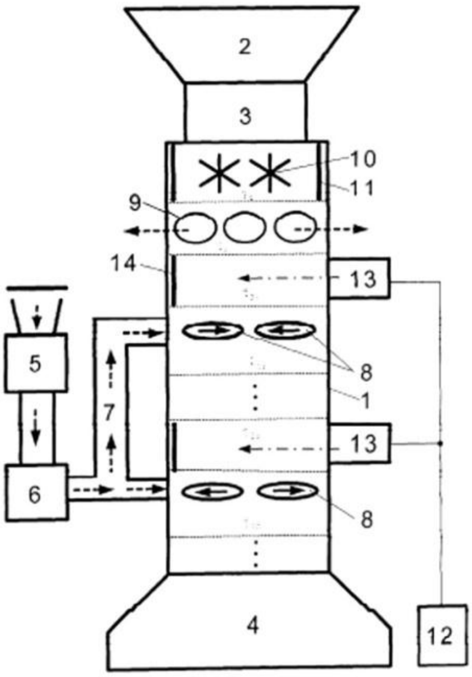
Figure 2.12 - Unit scheme of microwave for drying bulk materials according to the patent [45]

The unit works as follows. In drying chamber 1, the material is heated by microwave energy generated by microwave generators 9 and falls under its own weight to discharge device 3 where it is discharged by means of auger 13 with drive 14 into hopper 5.

During the heating process, the material is blown by heated fan 6. At the same time, air is sucked under the casing above microwave generators 9 through adapter 17; the air is heated by cooling the microwave generators, and is fed into draft chamber 16 and into the material through perforation. Part of the fan-pumped air is withdrawn by junction 12 and enters injector 10 where it performs two functions: creates vacuum in the suction chamber that improves the blowing-out, and twists the air in cyclone 11, improving separation of solids.

The disadvantage of this grain dryer is large thickness of the grain layer, through which it is necessary to blow air. This creates uneven distribution of air currents along the layers and, accordingly, uneven drying of grain. There is also an uneven distribution of microwave field along the thickness of grain layer.

Microwave grain treatment promotes its disinfection from grain pests and mycotoxins [46,47]. The use of ultraviolet radiation together with microwave treatment allows more efficient disinfection of grain. One of the grain dryer variants is presented in the patent [48]



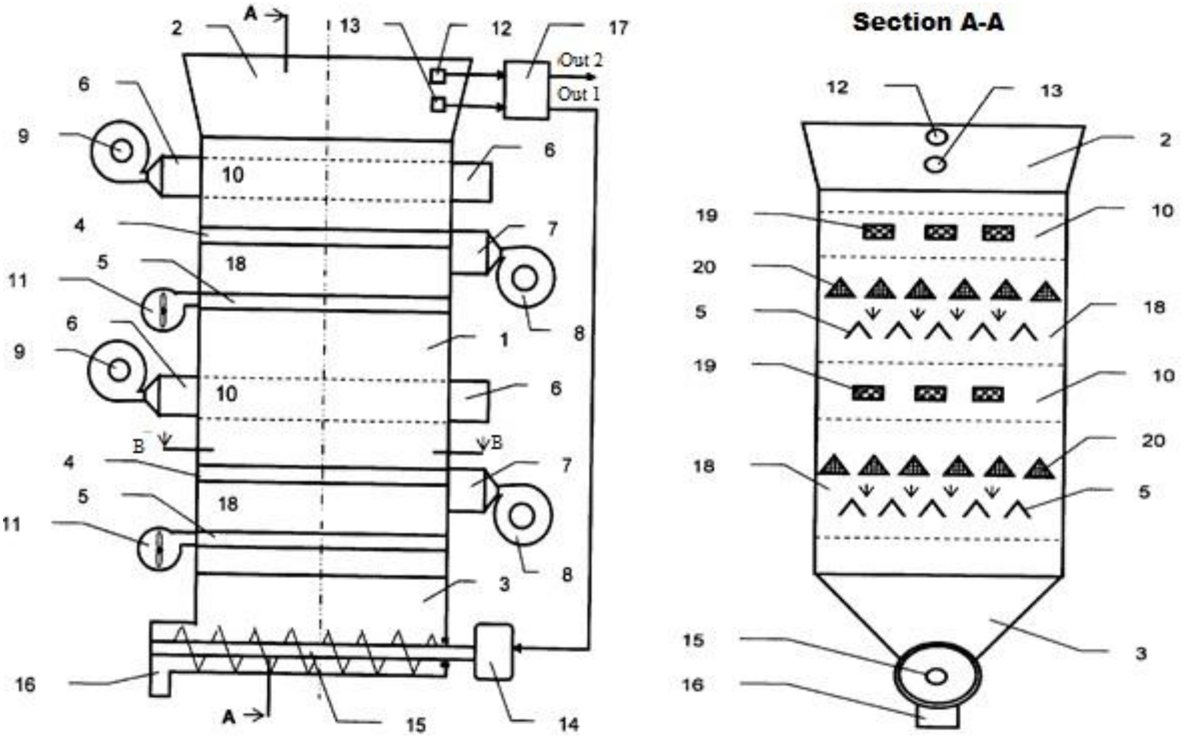
1- drying chamber; 2 - grain charge hopper; 3 - grain meter; 4 - grain discharge hopper; 5 - drying agent pump; 6 - preheating exchanger of drying agent; 7 - pump channel of drying agent; 8 - pump inputs of drying agent into the drying chamber; 9 – outlets of the used drying agent; 10 - sources of UV radiation; 11 - reflector of UV radiation; 12 - zone of drying agent removal; 13 - waveguide-slot exciters

Figure 2.13 - Construction of a column grain dryer according to the patent [48]

The column dryer works as follows. The grain is loaded into hopper 2 of the grain dryer, and through meter 3 under its own weight moves along drying chamber

1, sequentially through the zones. In drying chamber 1 grain is successively treated with UV radiation from UV radiation source (s) 10; in zone 11 grain is disinfected from microorganisms under the influence of UV radiation with a wavelength of 240-300 nm at a dose of 500-1000 W/cm; in zone 12 the used drying agent is removed.

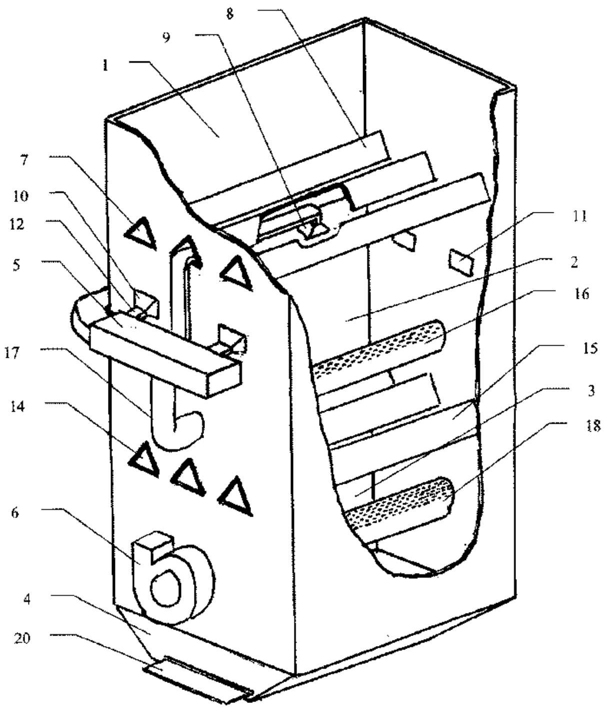
In convective microwaves grain dryers of column type, as well as in simple column grain dryers, there is a problem of a uniform distribution of drying agent. In addition, the distribution of microwave electromagnetic field inside the grain layer depends significantly on its moisture content. The higher the initial moisture level of the grain, the more limited penetration of the field into the grain layer. All variants of convective microwave grain dryers are designed to solve precisely these problems. Fig. 2.14. shows a variant that improves distribution of drying agent in the chamber [49].



1- drying chamber; 2 - charge hopper; 3 - discharge hopper; 4 - Λ-shaped ducts for the supply of drying agent; 5 - Λ-shaped relief ducts for drying agent; 6 - microwave generator; 7- heat generator; 8 - forcing fan for drying agent; 9 - forcing fan for cooling of magnetrons; 10 - zone of microwave heating; 11 - exhaust fan of drying agent; 12 -hopper high-level indicator; 13 -hopper low-level indicator; 14 - drive of the auger device for hopper discharge; 15 - auger device for hopper discharge; 16 - output of grain from the hopper discharge; 17 - control unit of grain flow; 18 - zone of moisture removal from drying agent; 19 - radio-transparent plugs of antennas for magnetrons of the microwave generator; 20 - inlets for the supply of drying agent; 21 - air supply duct of drying agent; 22 - air outlet duct of drying agent

Figure 2.14 - Scheme of grain dryer according to the patent [49]

The patent [50] presents a variant of drying chamber construction in which radiation source is located inside the grain layer (Fig. 2.15). In this case, the question of field distribution uniformity in the grain layer has been partially solved, but this problem remains unresolved.



1 - over-the-drier section of the drying chamber; 2 - heating section of the drying chamber; 3 - cooling section of the drying chamber; 4 - bottom narrowing part of the drying chamber; 5 - microwave generator; 6 - forcing fan; 7 - hot air outlets; 8 - ∧ -shaped ducts for the removal of hot air; 9 - vertical microwave emitting horn; 10 - horizontal microwave emitting horns; 11 - radio-transparent plugs of microwave-emitting horns; 12 - waveguides of horizontal microwave-emitting horns; 14 - cool air outlets; 15 - ∧ -shaped cold air ducts; 16 - perforated duct for hot air supply; 17 - adapter of hot air supply; 18 - perforated duct for cool air supply; 19 - adapter for cool air supply; 20 - shielding casing of magnetrons

Figure 2.15 - Section of the unit drying chamber for drying bulk materials according to the patent [50]

This unit for drying bulk materials consists of a vertical rectangular drying chamber, functionally divided into over-the-drier section 1, heating section 2 and cooling section 3 with bottom narrowing part 4, microwave generator 5, forcing fan 6, charge and discharge device (not shown on fig. .1). Between over-the-drier section 1 and heating section 2, between the holes 7 horizontal ∧ -shaped outlet ducts for hot air 8 are installed at the ends of the drying chamber; one of these ducts is oriented along the horizontal axis of symmetry and has vertical microwave-emitting horn 9 with radio-transparent plug, that looks inside the drying chamber; and radiation axis coincides with vertical axis of the drying chamber symmetry. On the opposite walls of heating section 2, horizontal microwave-emitting horns 10 are symmetrically positioned in relation to the vertical and horizontal axes, the openings of which are plugged by radio-transparent plugs 11 and look into the chamber, and radiation axes are located in one horizontal plane. Every microwave-emitting horn 10 via a standard waveguide 12 is connected to corresponding magnetron 13 with an individual power source. The microwave generator 5 includes several (more than 2) magnetrons 13. Between heating section 2 and cooling section 3, between holes 14, there are ∧ - shaped ducts 15 for cold air at the ends of the drying chamber. The heating section 2 is provided with a supply perforated duct 16 connected to hot air supply adapter 17. Cooling section 3 is provided with supply perforated duct 18, which is connected to forcing fan 6 via adapter 19, each magnetron 13 is placed in shielding housing 20. Magnetrons 13 are provided with a forced air cooling system, the outlet of which is connected to hot air supply adapter 17.

The main designer of microwave units in AST company B.G.Smirnov, presented several options for the implementation of such units. Fig. 2.16. shows a variant of a hopper microwave convection dryer.

Figure 2.16 - Hopper type microwave convective grain dryer produced by AST company

The dryer is structurally divided into two elements: microwave activation zone and grain drying hopper. First grain passes through the microwave activation zone, where, due to the microwave field action, moisture moves from the center of grain to its surface. Then grain is sent to the hopper, where convective drying takes place.

Figure 2.17 shows two variants for the use of a column microwave convective grain dryer.

a) b)

Figure 2.17 - Column microwave convective grain dryer.
a) for storage and b) indoors of ZAV

The column variant of a microwave-convective grain dryer is attractive for its small dimensions, which makes it possible to place them indoors of ZAV, combining the post-harvest cleaning process with grain drying.

The effect of the microwave field on the grain leads to a more intense heating of those parts of the grain that have higher moisture content. Unlike convective drying, where a part of the grain that is closer to the drying agent is heated to a higher degree. Thus, in microwave convective drying, the temperature and humidity gradients look in one direction. This increases drying efficiency. However, during the drying process, the moisture distribution in grain and in the grain layer does not remain constant. Therefore, distribution of the microwave field and, consequently, grain heating temperature also changes during the drying process.

These features must be taken into account when designing the constructions of microwave convective grain dryers. Therefore, it is advisable to consider the possibility of microwave field distribution modeling in the drying zone of different constructions. From these positions, the processes of heating / cooling grain in the microwave active zones of grain dryers are very interesting. The material presented in the second chapter is devoted to these studies.

CHAPTER 3 THEORETICAL STUDIES OF GRAIN HEATING AND COOLING IN THE PERIODICAL INFLUENCE OF THE MICROWAVE FIELD

3.1 Distribution of moisture in grain after harvesting

Anatomically grain is divided into three main parts: endosperm, embryo and surrounding covers, which differ sharply between each other in structure and properties. Paleaceous crops (rice, oats, barley) are covered with floral paleola or film [50].

The distribution of chemical substances by anatomical parts of wheat grain is given in Table 3.1.

Table 3.1 - Content of major chemical components in anatomical parts of grain,%

Anatomical parts of grain	Chemical components					Ash content
	protein	Starch	crude fiber	pentosans	lipids	
Fruit coat	5.0-7.6	-	20.5	27.5	1.0	3.4-4.3
Seed cover	12.0-19.5	-	1.0-1.2	13.8-36.0	0-0.2	12.6-20.0
Aleurone layer	18.0	-	-	-	-	14.4-17.2
Endosperm	12.9	78.8	0.15	2.7	0.7	0.45
Embryo with scutellum	24.3-41.3	-	2.46	9.7	15.0	5.35-6.32

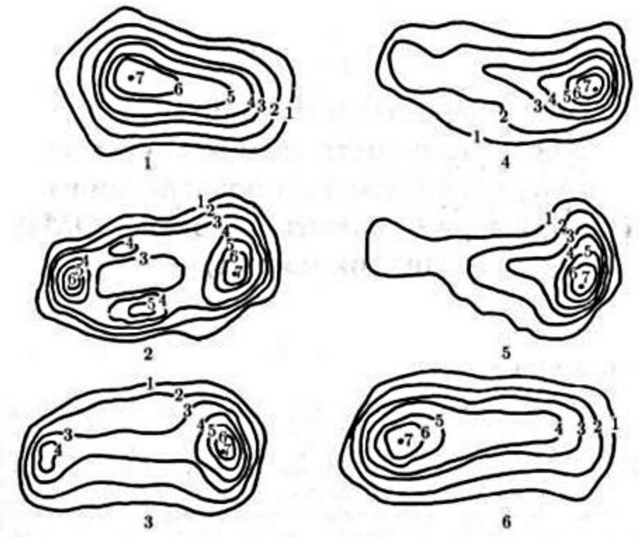
The complex shape of grains, structural features and chemical composition of covers, embryo and endosperm determine development of external heat and mass transfer processes, and internal transfer of moisture, heat and biologically important substances. Each grain part plays a special role in heat and moisture transfer processes.

Capillaries are essential for the processes of heat and moisture exchange; these are the channels through which moisture is transferred in grains. The grain lacks macrocapillaries, i.e. capillaries with a radius of more than 10-5 cm [50]. Such capillaries appear at a moisture content above 40%. The studies carried out [50] show that grain endosperm does not have macrocapillaries, but instead, grain have intermolecular gaps. Therefore, endosperm is a dense body, a colloidal (quasi-capillary porous) body. Macrocapillaries and pores are present only in the fruit coats of the grain, the empty cells of which have a large capacity. There are also significant gaps between individual groups of cells in the tubular layer, as well as between these cells and the seed coat. The gap dimensions exceed the value of $1 \cdot 10^5$ cm. Therefore, the fruit coats refer to capillary-porous body.

Such difference in the structure of endosperm and external coats significantly affects the processes of grain moistening and dehydration. The single grain should be considered as a complex compound body of the "ball-in-ball" type.

Wheat and buckwheat germ with relative air humidity
<55% is less hydrophylic than endosperm. However, with a higher humidity of the atmosphere, germ's moisture content changes much faster than other anatomical parts. Thus, at a relative humidity of 90%, a buckwheat germ sorbs 40% of moisture, and kernel 32%; for wheat these values are 50 and 30%, respectively [50]. Thus, an embryo absorbs moisture from air in the conditions of high atmospheric humidity. Similar processes occur when the grain is moistened, which must be taken into account in the technological processes before microwave treatment.

The data on the role of water and its state in biological processes have been obtained by studying the features of the water regime at various stages of wheat maturing by using spin echo NMR, ESR tomography, luminescent and other methods [51]. The obtained experimental data show that during the seed maturing moisture content is decreased , as well as the volume of the liquid phase within seeds. The distribution of moisture within the limits of the seeds significantly depends on the ripening stage (harvesting date) and weather conditions during harvesting. Fig. 3.1 shows two-dimensional image of the spin probe distribution in wheat seeds that have been harvested and dried at different ripening stages.



1 - in 14 days after flowering; 2 - in 19 days; 3 - in 27 days;
4 - in 32 days; 5 - in 36 days; 6 - in 36 days (after the rain) [52]

Figure 3.1 - Two-dimensional image of the spin probe distribution in wheat seeds that have been harvested and dried at different ripening stages.

It can be seen from the figure that precipitations, even for the matured wheat grain, significantly influence the pattern of moisture distribution in grain after drying.

The presented material allows us to draw the following conclusions:

- peculiarities of the grain anatomical structure predetermine unequal distribution of moisture in its parts;

- significant difference in the presence of capillaries in different parts of grain will significantly affect the movement within it;

- distribution of moisture within grain is influenced by harvesting time and weather conditions during the harvesting season;

- both in the case of artificial moistening of grain and with its natural drying, embryo has the greatest moisture, but not the grain surface, which has a considerable importance in microwave treatment of the grain layer.

3.2 Mathematical description of heat transfer in grain when exposed to a microwave field

A lot of factors influence heat and moisture exchange in the grain layer. Description of this process is quite difficult. Therefore, similarity theory is actively used for these purposes [53]. Yegorov [50] had carried out research on the effect of grain treatment regimes for absolute values of the Fourier exchange mass number Fo_m. In all cases the values of number Fo_m do not exceed 0,1. This means that the process of internal transfer of moisture, both during grain moistening and drying, lies entirely in the non-stationary area. In this case, a continuous change in moisture content and binding energy at each point of the body is observed; thermodynamic characteristics of the material are also changing. In this connection, an exact mathematical analysis of the internal moisture transfer process in grain is completely impossible.

The results of the studies on the dependence of Lykov Lu number on the moisture content during drying process are also reflected here. During the drying process, the values of the Lykov number quickly decreased and then remained unchanged, reaching $0,5 \cdot 10^{-4} \ldots 1,0 \cdot 10^{-4}$. This has allowed Yegorov to conclude that thermal and moisture conductivity does not play an important role in the transfer of internal moisture. Therefore, in case of drying and hydrothermal treatment, the processes of heat and moisture transfer can be considered separately.

In contrast with convective heating under the action of microwave field the kernels that are more moistened have the highest temperature. Due to intense heating of the grain wetter zones the water vapor pressure increases[54], moisture moves to the less heated sections of the grain. Therefore, in our opinion, to describe microwave heating, it is necessary to take into account the movement of moisture inside of grain.

Before we turn to the study of temperature variation patterns in various grain zones, it should be noted that the researches in this direction was carried out in the 80's ... 90's of the last century at the Academic school of Academician I.F. Borodin in MSAU named after V.P. Goryachkin. S.V. Vendin had obtained dependencies that make it possible to determine temperature at the desired point of grain under microwave exposure [55]. These dependencies can be successfully used for the calculation of processes in which the moisture distribution over grain is unimportant. Some techniques allow grain damping. The distribution of moisture over grains can be very important in the case of grain disinfection. Freshly harvested grain and grain after storage can both be disinfected. Therefore, obtaining mathematical dependencies of the temperature variation in different grain parts, depending on their moisture content, under the influence of microwave field is an urgent research objective, the solution of which will improve the technology of grain disinfection in microwave field.

3.2.1 Specifying the model for distribution of moisture in grain

Before proceeding to the development of mathematical model for the temperature change in grain during the exposure to microwave field, let us once again specify the features of the technological process for grain disinfection. First of all, we are talking about post-harvest treatment of the grain coming into the storage. Grain can be of conditional humidity or require drying (16 ... 18%), but in a microwave active zone the effect of electromagnetic field is provided without air exchange.

The system of differential heat and moisture exchange equations under microwave exposure has the following form [56]:

$$
\begin{cases}
\dfrac{\partial \theta}{\partial \tau} = a\dfrac{\partial^2 \theta}{\partial R^2} + \dfrac{\varepsilon}{cr'}\dfrac{\partial u}{\partial \tau} + \dfrac{Q_v}{c\rho_o}; & (3.1)\\
\dfrac{\partial u}{\partial \tau} = a_m\dfrac{\partial^2 u}{\partial R^2} + a_m\delta_2\dfrac{\partial^2 \theta}{\partial R^2} + \varepsilon\dfrac{\partial u}{\partial \tau}; & (3.2)\\
\dfrac{\partial P}{\partial \tau} = a_p\dfrac{\partial P^2}{\partial R^2} + \dfrac{\varepsilon}{c_v}\dfrac{\partial u}{\partial r}. & (3.3)
\end{cases}
$$

where a - coefficient of thermal diffusivity, m^2/s; ε – coefficient of phase transformation liquid-vapor; c – specific heat of grain, kJ/kg°C; r' –specific heat of vaporization, kJ / kg; Q_v– specific power dissipated in the dielectric under the influence of microwave field, W/m^3; ρ_0 – density of grain dry matter, kg/m^3; a_{m_2} – coefficient of fluid diffusion, m^2/s; δ_2- relative coefficient of thermal diffusion; P-

excess pressure in the sample, Pa; c_v- body capacity relative to moist air, $c_v = \frac{1}{P_н}, \frac{1}{\text{Pa}}$; $P_н$- vapor pressure of the material at a given moisture content, Pa; Θ - temperature of grain, °C; a_p – coefficient of convective vapor diffusion, m^2/s.

During the microwave grain treatment for the purpose of its disinfection, the main objective is temperature effect on the grain. It is not intended to blow the grain layer with air in the process of treatment, because reducing the grain moisture content is not an objective. Since grain is treated in a dense layer, it can be assumed that moisture is released into an intergrain space under microwave influence; and then this moisture is absorbed by grain. Since moisture content of all grains is the same, there is no moisture exchange between them. Therefore, we assume that in the process of grain microwave treatment the moisture exchange with the surrounding medium does not occur. Consequently, in equations (3.1), (3.2), (3.3), we can take the change rate of humidity as equal to zero

$$\frac{\partial u}{\partial \tau} = 0.$$

Then the system of equations will look like this.

$$\frac{\partial \theta}{\partial \tau} = a\frac{\partial^2 \theta}{\partial R^2} + \frac{Q_v}{c\rho_o} \qquad (3.4);$$

$$\frac{\partial^2 U}{\partial R^2} = \delta_2 \frac{\partial^2 \theta}{\partial R^2} \qquad (3.5);$$

$$\frac{\partial P}{\partial \tau} = a_\text{p} \frac{\partial P^2}{\partial R^2} \qquad (3.6).$$

Equation (3.4) shows that the change rate of grain temperature depends on the change rate of temperature gradient in grain and on a specific power released, which in its turn depends on dielectric permittivity of grain. Since moisture content of grain is unevenly distributed, the heating rate of grain various sections will also be different.

Equation (3.5) shows that the rate of temperature gradient change is proportional to the change rate of moisture content gradient. Equation (3.6) shows that the change rate in vapor pressure of grain moisture is proportional to the change rate of the pressure gradient. In our calculations, this equation is not needed.

If in equation (3.4) the term $(\partial\hat{}2\,\theta)/(\partial R\hat{}2)$ is replaced by its value from equation (3.5), then we obtain the following control system:

$$\frac{\partial \theta}{\partial \tau} = \mathrm{a}\frac{\partial^2 \theta}{\partial R^2} + \frac{Q_v}{c\rho_o} \qquad (3.4);$$

$$\frac{\partial \theta}{\partial \tau} = -\frac{a}{\delta_2}\frac{\partial^2 U}{\partial R^2} + \frac{Q_v}{c\rho_o} \quad (3.7)$$

This system of equations describes dependence of the grain temperature change rate on the rate of change in the temperature and humidity gradients.

Equation (3.4) is a heat equation, describing the change in temperature at each point of the object when energy is supplied from outside [57,58,59].

Heat conductivity equation belongs to the tasks of parabolic type in mathematical physics [60, 61]. It is necessary to know temperature distribution at the initial moment of time (i.e. initial conditions) in order to describe dynamics of the temperature field in the body with the help of heat equation, In addition, it is required to know thermal regime on the surface of the body, i.e. set the boundary conditions at all points of the body surface at any time.

To set initial and boundary conditions, we shall consider the properties of grain coming into a microwave field in more detail.

The processes of grain heating and moving moisture essentially depend on the moisture content of the various grain layers. Previously, it has been shown that when grain matures, distribution of moisture in grain depends on the timing of harvesting and partially on the grain volume. We will assume that the seeds have a round shape, then distribution of moisture in grain after harvesting can be conditionally represented in the form of a figure.

1-7 - relative zones of grain with the same humidity. Zone with a large number corresponds high humidity: $W_7 > W_6 > W_5 > W_4 > W_3 > W_2 > W_1$.

When exposed to the microwave field, the grain parts heating will occur in accordance with distribution of moisture. Consequently, the outer grain layers will be heated to a lesser extent, the inner layers - to a greater extent.

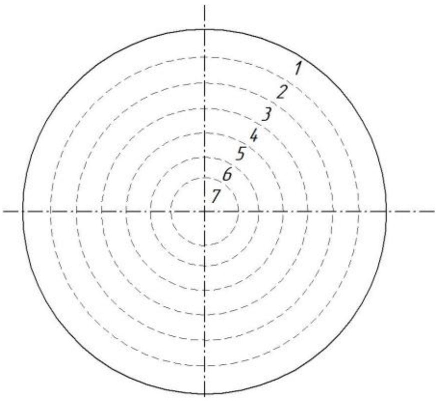


Figure 3.2 - Relative moisture distribution scheme in the grain after harvesting

As it was noted earlier, during the grain maturing, the form of moisture binding is changing in the grain, which can lead to a change in dielectric permittivity of water in different parts of the grain volume. Therefore, the difference in the heating of grain parts will depend on the maturity degree of the seeds. It is very difficult to obtain objective information about dielectric properties of water in various parts of grain. So we accept the third assumption that these properties are constant in each of grain zones, where we consider the constant moisture.

Taking into account the mentioned above, the grain heating scheme under the action of microwave field can be represented as follows. Since the humidity of grain layers decreases from center to surface, the maximum temperature will also be at the center of grain. As you move away from the center, the temperature will decrease. The moisture transfer in grain during microwave heating occurs due to the effect of the temperature gradient $\theta_1 > \theta_2 > \theta_3 > \theta_4 > \theta_5 > \theta_6 > \theta_7$, pressure gradient $P_1 > P_2 > P_3 > P_4 > P_5 > P_6 > P_7$, where $\theta_{1 \ldots}$ θ_7 is the temperature, in the grain zones, which has arisen under the action of microwave field.

The movement of vapor from the core center through the capillaries leads to faster heating of the grain remaining parts, which have a lower dielectric permittivity. If the exposure time of the microwave field is sufficient, the temperatures of the center and the surface of the grain should be leveled. In this case it is necessary to take into account that the heating temperature of the grain, especially its center, can exceed the safe value. Therefore, it is necessary to control heating temperature of the grain and to limit the exposure time of the microwave field. As a result, it may turn out that the temperature of the whole grain does not level. Since during grain disinfection we are interested in the temperature of the layers located closer to the surface, it is possible that it will be necessary to repeat exposure to microwave field several times to ensure the required temperature and exposure in the surface layers of grain.

It is necessary to specify the initial and boundary conditions, which depend on the structure of the grain to perform the calculation of grain heating under the microwave field. In order to adequately reflect mathematical model processes occurring in the grain, it is necessary to take into account and describe all the main interactions. It seems that there is no need to divide the grain into seven moisture zones, as shown in Fig. 3.1. Considering small size of grain for the mathematical model, description of the grain heating process in three zones will be enough: central, middle and external. Then, to simplify the problem, we assume that grain has shape of a sphere and is divided into three zones, where humidity is uniformly distributed (Fig. 3.3).

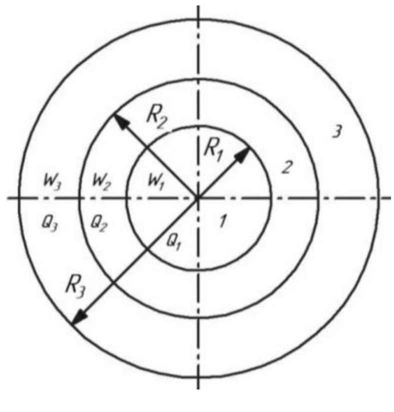


Figure 3.3 - Relative representation of the grain and separation into three moisture zones

3.2.2 Recording boundary-value problems for grain processing in the microwave field

Since we divided grain into three zones in accordance with moisture content, then in the mathematical models of the temperature change process under microwave exposure the accepted conditions should be contained. This should be reflected in the record of the boundary conditions.

The boundary conditions in heat conduction problems can be specified in various ways [62, 63, 64].

Boundary conditions of the first kind is when at each point of the body surface temperature is assigned.

Boundary conditions of the second kind is when the heat flux is assigned on the surface of the body.

Boundary conditions of the third kind is when the thermal regime on the surface of the body is described, corresponding to convective heat transfer according to Newton's law with the surrounding environment.

Boundary conditions of the fourth kind is when boundary conditions of conjugation are used in describing the temperature fields in multilayer bodies and covers on the contact surface of two bodies. For an ideal thermal contact, these conditions are the equality of temperatures and heat fluxes on the contact surface.

It follows from the above boundary conditions for grain that in the statement of the heat transfer problem the several methods of boundary conditions are used. Thus, for a circle, inner ring, and inner surface of an external ring, boundary conditions of the fourth kind are used. Therefore $\theta_{b\,1.1}$ should be equal to

$$\theta_{b1.2} = \frac{\left(\theta_2 = f(\tau)\right) + \left(\theta_3 = f(\tau)\right)}{2}.$$

Similar

$$\theta_{b2.1} = \theta_{b2.2} = \frac{\left(\theta_2 = f(\tau)\right) + \left(\theta_3 = f(\tau)\right)}{2}.$$

For the external ring, which conducts convective heat exchange with the external environment, the boundary conditions of the third kind are used.

Previously, it has been assumed that in each of three zones the grain has the same thermal properties within a zone. Therefore, if we select round cylinder along grain's central axis (Figure 3.4), then the temperature change in the sections of this rod will be similar to the temperature changes in any point of the rings.

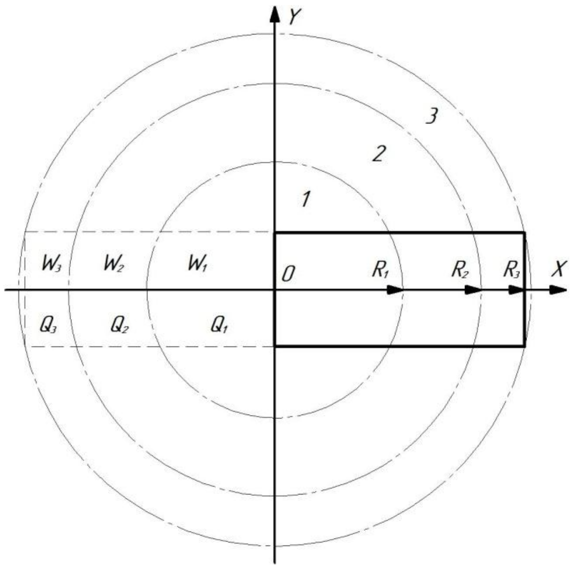


Figure 3.4 - Cylinder from grain central axis

Taking into account this symmetry, the heat conduction problem for the grain can be reduced to solving the heat conduction problem for a rod. Since the temperature around the rod will always be equal to the temperature inside of the rod, therefore, there will be no heat exchange from the side surface of the rod. Consequently, the problem is similar to the problem for a rod with an isolated surface.

Let us write the boundary-value problem with initial and boundary conditions.

For microwave heating. Since the rod is symmetric with respect to the reference point, we will perform calculation only relative to the right side.

Formulation of the boundary value problem for the first section of the rod will be as follows. Get the equation of temperature change with the rod length R_1, with a heat-insulated side surface and a left cone (x=0), if the initial temperature of the rod is Θ_0. Inside the rod there is a uniformly distributed source of energy Q_{v1}. At the right edge of the rod, the temperature varies according to the law

$$\theta_{b1} = \frac{\theta_1(\tau) + \theta_2(\tau)}{2};$$

The boundary-value problem for the first section of the rod will have the following form.

$$
\begin{cases}
\dfrac{\partial\theta_1}{\partial\tau} = a_1^2\dfrac{\partial^2\theta_1}{\partial x^2} + \dfrac{Q_{v1}}{\rho_1 c_1}, \qquad 0 < x < 2R_1, \qquad 0 < \tau < +\infty \\
\theta_1(x,0) = \Theta_{0,} \qquad\qquad 0 < x < 2R_1, \\
\theta_1(R_1,\tau) = \dfrac{\theta_1(\tau) + \theta_2(\tau)}{2} = \theta_{b1}(\tau), \\
\lambda S\dfrac{\partial\theta_1}{\partial x}(0,\tau) = 0, \qquad\qquad 0 < \tau < +\infty,
\end{cases}
\tag{3.8}
$$

where a_1- coefficient of thermal conductivity of the central part (ball/grain m^2/s), p_1 –density of grain area, kg / m^3; C_1 – Specific heat of the grain section, kJ / kg · K; Q_{v1}- Specific power released in the grain section under the action of microwave field, W / m^3; S - cross-section of the rod, m^2; q– heat flux.

For the second section of the rod the boundary value problem will have the following form.

$$
\begin{cases}
\dfrac{\partial\theta_2}{\partial\tau} = a_2\dfrac{\partial^2\theta_2}{\partial x^2} + \dfrac{Q_{v2}}{\rho_2 c_2}, \qquad 2R_1 < x < R_2, \quad 0 < \tau < +\infty, \\
\theta_2(x,0) = \Theta_0, \qquad\qquad 2R_1 < x < R_2, \\
\theta_2(R_1,\tau) = \theta_{b1}(\tau), \theta_2(R_2,\tau) = \theta_{b2}(\tau), \qquad 0 < \tau < +\infty
\end{cases}
\tag{3.9}
$$

For the third section of the grain, taking into account the fact that convective heat transfer occurs from the right end according to Newton's law, the boundary value problem will have the following form.

$$
\begin{cases}
\dfrac{\partial\theta_3}{\partial\tau} = a_3\dfrac{\partial^2\theta_3}{\partial x^2} + \dfrac{Q_{v3}}{\rho_3 c_3}, \qquad R_2 < x < R_3, 0 < \tau < +\infty, \\
\theta_3(x,0) = \Theta_0, \quad R_2 < x < R_3, \\
\theta_3(R_2,\tau) = \theta_{b2}(\tau), \qquad \theta_3(R_3,\tau) = h[\theta_3(R_3,\tau) - T_{м3}(\tau)], \qquad 0 < \tau < +\infty
\end{cases}
\tag{3.10}
$$

where $h = \alpha/\lambda$, 1/m; α -coefficient of heat exchange between the surface of the rod and the environment with the temperature T_{ig} (τ),W/m2·K; λ – coefficient of grain section internal thermal conductivity W/m·K; T_{ig} (τ) – temperature of intergrain space, ^{0}C.

Without the influence of the microwave field. After the microwave field stops, grain starts cooling down. The boundary value problems for each part will have the following form.

For the first section.

$$\begin{cases} \dfrac{\partial \theta_1}{\partial \tau} = a_1^2 \dfrac{\partial^2 \theta_1}{\partial x^2}, \qquad 0 < x < 2R_1 \\ \theta_1(x,0) = \theta_{1к,} \qquad 0 < x < 2R_1, \\ \lambda S \dfrac{\partial \theta_1}{\partial x}(0,\tau) = 0, \qquad \theta_1(R_1,\tau) = \theta_{b1}(\tau), \end{cases} \tag{3.11}$$

For the second section.

$$\begin{cases} \dfrac{\partial \theta_2}{\partial \tau} = a_2^2 \dfrac{\partial^2 \theta_2}{\partial x^2}, \qquad 2R_1 < x < R_2 \\ \theta_2(x,0) = \theta_{2к}, \qquad \theta_2(R_2,\tau) = \theta_{b2}(\tau), 0 < \tau < +\infty \end{cases} \tag{3.12}$$

For the third section.

$$\begin{cases} \dfrac{\partial \theta_3}{\partial \tau} = a_3^2 \dfrac{\partial^2 \theta_3}{\partial x^2}, \qquad R_2 < x < R_3, \ \ 0 < \tau < +\infty, \\ \theta_3(x,0) = \theta_{3к,} \qquad R_2 < x < R_3, \\ \theta_3(R_2,\tau) = \theta_{b2}(\tau), \quad \theta_3(R_3,\tau) = -h[\theta_3(R_3,\tau) - T_{м3}(\tau)], 0 < \tau < +\infty, \end{cases} \tag{3.13}$$

where $\theta_{1к}$, $\theta_{2к}$, $\theta_{3к}$, is the temperature of the first, second and third sections of the grain, respectively, after the microwave field stops.

With further, cyclic, continuation of "heating" - "cooling" process, the final temperatures of the "heating" or "cooling" stages must be taken as initial conditions for the next stage. Thus, in order to obtain regularities in the grain temperature change in the cyclic heating-cooling process, it is necessary to solve six boundary value problems (3.8) ... (3.13).

3.2.3 The solution of boundary value problems for microwave heating

To solve boundary value problems, we use the Laplace integral transformation method. The transformation of partial derivatives is carried out according to this rule [67, 68, 69]: if $\theta = \theta(x,\tau)$ and Laplace transformation is carried with variable τ ($\tau \geq 0$), then, set $L[\theta(x,\tau)] \equiv \Theta(x,p) = \int_0^{+\infty} e^{-p\tau}\Theta(x,\tau)d\tau$, it is possible (by integration by parts) to establish the relation

$$L\left[\frac{\partial \theta}{\partial \tau}\right] = p\Theta(x,p) - \theta(x,0),$$

$$L\left[\frac{\partial^2 \theta}{\partial \tau^2}\right] = p^2\Theta(x,p) - p\theta(x,0) - \frac{\partial \theta}{\partial \tau}(x,0),$$

$$L\left[\frac{\partial \theta}{\partial x}\right] = \frac{\partial \Theta}{\partial x}(x,p), \qquad \left[\frac{\partial^2 \theta}{\partial x^2}\right] = \frac{\partial^2 \Theta}{\partial x^2}(x,p)$$

Under certain conditions on the function Θ(x,τ), where L is a Laplace operator; Θ (x,p) – variable Θ after performing the direct Laplace transformation in time (τ).

To simplify the procedure for writing equation further on, instead of the partial derivatives symbol, let us use the index of the variable by which the derivative is taken. For example

$$\frac{\partial \theta}{\partial \tau} = \theta_\tau; \ \frac{\partial^2 \theta}{\partial \tau^2} = \theta_{\tau\tau.}$$

We proceed to the solution of the first boundary-value problem

$$\begin{cases} \theta_\tau = a^2\,\theta_{xx} + P_{sp1}, \ \ 0 < x < 2R_1, \ \ 0 < \tau < +\infty \\ \theta(x,0) = \Theta_{01}, \ \ 0 < x < 2R_1 \\ \lambda S \theta_x\,(0,\tau) = 0, \ \ \theta(R_1,\tau) = \theta_{b1}(\tau) \end{cases} \tag{3.14}$$

where

$$P_{\text{sp}1} = \frac{Q_{v1}}{p_1 c_1}, \quad \Theta = \Theta_1$$

If $\lambda S \theta_x(0,\tau) = 0$, then $\theta_x(0,\tau) = 0$., let's take this into account during the calculations.

To solve this problem, we use the Laplace transformation with respect to the variable τ. Taking into account the properties of this transformation [67], we have:

$$\theta(x,\tau) \doteqdot \Theta(x,\text{p}), \ \ \theta_\tau(x,\tau) \doteqdot \text{p}\Theta(x,\text{p}) - \theta(x,0),$$

$$\theta_x(x,\tau) \doteqdot \Theta_x(x,\text{p}), \qquad \theta_{xx}(x,\text{p}) \doteqdot \Theta_{\text{p}xx}(x,\text{p}).$$

It follows from the problem (2.14) that

$\text{L}[\theta_\tau] = a^2 L[\theta_{xx}] + P_{sp1}$,

$with\ \theta(x,\tau) = \Theta_{01}$we will get

$a^2\Theta_{xx}(x,p) - \text{p}\Theta_{\text{p}}(x,p) + \Theta_{01} + \frac{1}{P} P_{sp1} = 0$

As a result, an ordinary differential equation of the second order in the variable x (in this equation p plays the role of the parameter). The initial condition $\theta(x,0)$ уже вошло in the equation, which is a positive moment of this method for solving partial differential equations.

We take the Laplace transformation to boundary conditions. Let us set $\theta_{b1}(\tau)\ as$ G(τ), then L $[G(\tau)] \doteqdot G(p)$, and L $[\theta_x(0,\tau)] = \Theta(0,p)$

After performing the Laplace transformation, the boundary value problem is written in the following form:

$$\begin{cases} a^2\Theta_{1xx}(x,\mathrm{p}) - p\Theta(x,\tau) + P_{sp1} + \Theta_{01} = 0, \quad 0 < x < 2R_1 \\ \Theta_1(0,p) = G_1(p), \quad \Theta_1(2R_1,p) = G_1(p), \quad 0 < \tau < +\infty \end{cases} \tag{3.15}$$

The solution of the differential equation, taking into account the boundary conditions $\Theta(0,\mathrm{p})$ and $\Theta(2R_1,\mathrm{p})$, is carried out using MATLAB software [68, 69].

The solution has the following form:

$$\Theta_1(x,p) =$$

$$\frac{P_{sp1}+\Theta_{01}p}{p^2} + \frac{e^{+\frac{\sqrt{p}x}{a}}(P_{sp1}+\Theta_{01}p-G_1(p)p^2-P_{sp1}e^{-\frac{\sqrt{p}2R_1}{a}}-\Theta_{01}pe^{-\frac{\sqrt{p}2R_1}{a}}+G_1(p)p^2e^{-\frac{\sqrt{p}2R_1}{a}})}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})} -$$

$$\frac{e^{-\frac{\sqrt{p}x}{a}}(P_{sp1}+\Theta_{01}p-G_1(\mathrm{p})p^2-P_{sp1}e^{\frac{\sqrt{p}2R_1}{a}}-\Theta_{01}\mathrm{p}e^{\frac{\sqrt{p}2R_1}{a}}+G_1(\mathrm{p})p^2e^{\frac{\sqrt{p}2R_1}{a}})}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})} \tag{3.16}$$

Let us set the second and third fractions to the common denominator, remove parentheses, and perform the transformations

$$\frac{e^{\frac{\sqrt{p}x}{a}}+P_{sp1}e^{\frac{\sqrt{p}x}{a}}\Theta_{01}\mathrm{p}-G_1(\mathrm{p})p^2e^{\frac{\sqrt{p}x}{a}}-P_{sp1}e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}x}{a}}-\Theta_{01}pe^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}x}{a}}+G_1(\mathrm{p})p^2e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}x}{a}}}{p^2\left(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}\right)}$$

$$-\frac{e^{-\frac{\sqrt{p}x}{a}}P_{sp1}-\Theta_{01}pe^{-\frac{\sqrt{p}x}{a}}+G_1(\mathrm{p})p^2e^{-\frac{\sqrt{p}x}{a}}+P_{sp1}e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}x}{a}}+\Theta_{01}pe^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}x}{a}}}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})}$$

$$-\frac{G_1(\mathrm{p})p^2e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}\mathrm{x}}{a}}}{\mathrm{p}^2\left(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}\right)}. \tag{3.17}$$

After performing the transformation we obtain:

$$\frac{P_{sp1}\left(e^{\frac{\sqrt{p}\,\mathrm{x}}{a}}-e^{-\frac{\sqrt{p}\,\mathrm{x}}{\mathrm{a}}}\right)+\Theta_{01}p\left(e^{\frac{\sqrt{p}\,\mathrm{x}}{a}}-e^{-\frac{\sqrt{p}\,\mathrm{x}}{\mathrm{a}}}\right)+G_1(\mathrm{p})p^2\left(e^{-\frac{\sqrt{p}\,x}{a}}-e^{\frac{\sqrt{p}\,x}{\mathrm{a}}}\right)+P_{sp1}\left(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}\,x}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}\,x}{\mathrm{a}}}\right)+}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})}$$

$$\frac{\Theta_{01}p\left(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}\,x}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}\,x}{\mathrm{a}}}\right)+G_1(\mathrm{p})p^2\left(e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}\,x}{\mathrm{a}}}-e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}\,x}{a}}\right)}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})} =$$

$$\frac{\left(e^{\frac{\sqrt{p}\,x}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}\right)(P_{sp1}-\Theta_{01}p)+G_1(p)p^2\left(e^{-\frac{\sqrt{p}\,x}{a}}-e^{\frac{\sqrt{p}\,x}{a}}\right)}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})}+$$

$$\frac{\left(e^{\frac{\sqrt{p}(2R_1-x)}{a}}-e^{-\frac{\sqrt{p}(2R_1-x)}{a}}\right)(P_{sp1}-\Theta_{01}p)+G_1(p)p^2(e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}})}{p^2(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}})}=$$

$$\frac{e^{\frac{\sqrt{p}\,x}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}}{(e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}\cdot\frac{P_{sp1}-\Theta_{01}p}{p^2}+\frac{e^{-\frac{\sqrt{p}\,x}{a}}-e^{\frac{\sqrt{p}\,x}{a}}}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}\cdot\frac{G_1(p)p^2}{p^2}+\frac{e^{\frac{\sqrt{p}(2R_1-x)}{a}}-e^{-\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}\cdot$$

$$\frac{P_{sp1}-\Theta_{01}p}{p^2}+\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}\cdot\frac{G_1(p)p^2}{p^2}. \tag{3.18}$$

We perform the transformations of the left cofactors of each equation term (3.18)

$$\frac{\left(e^{\frac{\sqrt{p}\,x}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}\right)}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}=\frac{2Sinh\,(\sqrt{p}\,\frac{x}{a})}{-2Sinh\,(\sqrt{p}\,\frac{2R_1}{a})}=-\frac{Sinh\left(\sqrt{p}\,\frac{x}{a}\right)}{Sinh\left(\sqrt{p}\,\frac{2R_1}{a}\right)}, \tag{3.18.1}$$

$$\frac{e^{\sqrt{p}x}-e^{\frac{\sqrt{p}x}{a}}}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}=\frac{-2Sinh\,(\sqrt{p}\frac{x}{a})}{-2Sinh\,(\sqrt{p}\frac{2R_1}{a})}=\frac{Sinh\left(\sqrt{p}\frac{x}{a}\right)}{Sinh\left(\sqrt{p}\frac{2R_1}{a}\right)} \tag{3.18.2}$$

$$\frac{l^{\frac{\sqrt{p}(2R_1-x)}{a}}-l^{-\frac{\sqrt{p}(2R_1-x)}{a}}}{l^{-\frac{\sqrt{p}2R_1}{a}}-l^{\frac{\sqrt{p}2R_1}{a}}}=\frac{2Sin\,h\,(\sqrt{p}\frac{2R_1-x}{a})}{-2Sin\,h\,(\sqrt{p}\frac{2R_1}{a})}=-\frac{Sin\,h\,(\sqrt{p}\frac{2R_1-x}{a})}{Sin\,h\,(\sqrt{p}\frac{2R_1}{a})} \tag{3.18.3}$$

$$\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{-\frac{\sqrt{p}2R_1}{a}}-e^{\frac{\sqrt{p}2R_1}{a}}}=\frac{-2Sinh\,(\sqrt{p}\frac{2R_1-x}{a})}{-2Sinh\,(\sqrt{p}\frac{2R_1}{a})}=\frac{Sinh\,(\sqrt{p}\frac{2R_1-x}{a})}{Sinh\,(\sqrt{p}\frac{2R_1}{a})} \tag{3.18.4}$$

Now we can perform a inverse Laplace transformation for the equation (3.18). The originals of the right cofactors of the equation terms (3.18) are easily found from the original tables. The originals of the left cofactors (3.18.1), (3.18.2), (3.18.3) and (3.18.4) can be found from the original table of the monograph: [70]

$$-\frac{Sinh\left(\sqrt{p}\frac{x}{a}\right)}{Sinh\left(\sqrt{p}\frac{2R_1}{a}\right)}=\frac{2\pi}{\frac{4R_1^2}{a^2}}\;Sin\left(\frac{\pi}{\frac{2R_1}{a}}x\right)e^{\frac{\pi^2}{\frac{4R_1^2}{a^2}}\tau}=-\frac{\pi}{2}\frac{a^2}{R_1^2}Sin(\frac{\pi}{2}\frac{a}{R_1}x)e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau},$$

$$\frac{Sinh\left(\sqrt{p}\frac{x}{a}\right)}{Sinh\left(\sqrt{p}\frac{2R_1}{a}\right)}=\frac{\pi}{2}\frac{a^2}{R_1^2}Sin(\frac{\pi}{2}\frac{a}{R_1}x)e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau},$$

$$-\frac{Sinh\left(\sqrt{\mathrm{p}}\frac{2R_1-x}{a}\right)}{Sinh\left(\sqrt{\mathrm{p}}\frac{2R_1}{a}\right)} = -\frac{\pi}{2}\frac{a^2}{R_1^2}Sin(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau},$$

$$\frac{Sinh\left(\sqrt{\mathrm{p}}\frac{2R_1-x}{a}\right)}{Sinh\left(\sqrt{\mathrm{p}}\frac{2R_1}{a}\right)} = \frac{\pi}{2}\frac{a^2}{R_1^2}Sin(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}.$$

The originals of each equation term (3.18) can be found using a convolution image.

Let us consider realization of the convolution image. The convolution of the functions f and g is a function which is denoted by $fg$and defined by [70] $(fg)(\tau) = \int_0^\tau f(\tau_1)\ g(\tau-\tau_1)d\tau_1$

We assume that for the equation terms (3.18) the left side cofactors will be as follows:

$$f_1(\tau_1) = -\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1},$$

$$f_2(\tau_1) = \frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1},$$

$$f_3(\tau_1) = -\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1},$$

$$f_4(\tau_1)=\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}.$$

The right parts will look like this:

$q_1(\tau-\tau_1) = P_{sp1}(\tau-\tau_1) - \Theta_{01},\ q_2(\tau-\tau_1) = G_1(\tau-\tau_1),$

$q_3(\tau-\tau_1) = P_{sp1}(\tau-\tau_1) + \Theta_{01},\ q_3(\tau-\tau_1) = G_1(\tau-\tau_1).$

Then the original of the equation (3.18) can be found as follows $(f_1q_1)(\tau) + (f_2q_2)(\tau) + (f_3q_3)(\tau) + (f_4q_4)(\tau) =$

$$-\int_0^\tau (P_{sp1}(\tau-\tau_1) - \Theta_{01})\cdot\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 +$$

$$\int_0^\tau G_1\,(\tau-\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 -$$

$$\int_0^\tau \left(P_{sp1}(\tau-\tau_1)+\Theta_{01}\right)\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 +$$

$$\int_0^\tau G_1(\tau-\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1$$

Integrating these equations, we obtain an original expression (3.18).

$$-P_{sp1}\frac{2}{\pi^3 a^2}\cdot Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\right) - \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right) + \int_0^\tau G_1(\tau-\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 - P_{sp1}\frac{2}{\pi^3 a^2}\cdot$$

$$Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\right) + \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right) +$$

$$\int_0^\tau G_1(\tau-\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}\,d\tau_1$$

Using the linearity property for (3.16), let us write the complete expression for $\Theta_1(x,\tau)$ under microwave heating

$$\Theta_1(x,\tau) = \Theta_{01} + P_{sp1}\tau + \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\left(Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right) - Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)\right) - P_{sp1}\frac{2}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\right)\left(Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right) + Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)\right) + \frac{\pi}{2}\frac{a^2}{R_1^2}\left(Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right) + Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)\right)\int_0^\tau G_1(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1. \quad (3.19)$$

For the central part ($x = R_1$) the expression will have the following form:

$$\Theta_1(R_1,\tau) = \Theta_{01} + P_{sp1}\tau + \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\left(1 - Sin\left(\frac{\pi}{2}a\right)\right) - P_{sp1}\frac{2}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\right)\left(Sin\left(\frac{\pi}{2}a\right)+1\right) + \frac{\pi}{2}\frac{a^2}{R_1^2}\left(Sin\left(\frac{\pi}{2}a\right)+1\right)\cdot\int_0^\tau G_1(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 \quad (3.20)$$

For the boundaries of the central part ($x = 2R_1$)

$$\Theta_1(2R_1,\tau) = \Theta_{01} + P_{sp1}\tau - \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)Sin(\pi a) - P_{уд1}\frac{2}{\pi^3 a^2}\cdot$$
$$\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)\right)\cdot Sin(\pi a) + \frac{\pi}{2}\frac{a^2}{R_1^2}\cdot Sin(\pi a)\int_0^\tau G_1(\tau -$$
$$\tau_1)\,e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1. \qquad (3.21)$$

Thus, there are three key expressions describing change in the temperature of grain central part during microwave heating. Expressions (3.19), (3.20) and (3.21) will be confirmed after finding the dependence $G_1(\tau_1)$.

Let us turn to the solution of the boundary value problem for the second section (3.9).

After performing the Laplace transformation, the boundary value problem is written in the following form:

$$\begin{cases} a^2\Theta_{2xx}(x,p) - p\Theta_2(x,p) + \dfrac{P_{sp2}}{p} + \Theta_{02} = 0, & 2R_1 < x < R_2 \\ \Theta_2(2R_1,p) = G_1(p), & \Theta_2(R_2,p) = G_2(p) \end{cases} \qquad (3.22)$$

The solution of the differential equation, taking into account the boundary conditions is carried out using MATLAB software [68]. As a result, we obtain the following equation.

$$\Theta_2(x,p) =$$
$$\frac{P_{sp2}+\Theta_{02}p}{p^2} -$$
$$\frac{e^{-\frac{\sqrt{p}\,x}{a}}(P_{sp2}e^{\frac{\sqrt{p}2R_1}{a}}+\Theta_{02}pe^{\frac{\sqrt{p}2R_1}{a}}-G_2(p)p^2e^{\frac{\sqrt{p}2R_1}{a}}-P_{sp2}e^{\frac{\sqrt{p}R_2}{a}}-\Theta_{02}pe^{\frac{\sqrt{p}R_2}{a}}+G_1(p)p^2e^{\frac{\sqrt{p}R_2}{a}}}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})} +$$
$$\frac{e^{\frac{\sqrt{p}\,x}{a}}(P_{sp2}e^{-\frac{\sqrt{p}2R_1}{a}}+\Theta_{02}pe^{-\frac{\sqrt{p}2R_1}{a}}-G_2(p)p^2e^{-\frac{\sqrt{p}2R_1}{a}})}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})} +$$
$$\frac{e^{\frac{\sqrt{p}\,x}{a}}\left(-P_{sp2}e^{-\frac{\sqrt{p}R_2}{a}}-\Theta_{02}pe^{-\frac{\sqrt{p}R_2}{a}}+G_1(p)p^2e^{-\frac{\sqrt{p}R_2}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}\,. \qquad (3.23)$$

Transforming the second and third fractions of (3.23), we obtain

$$-\frac{e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}2R_1}{a}}P_{sp2}-e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}P_{sp2}+e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}2R_1}{a}}\Theta_{02}p-e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\Theta_{02}p}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}$$

$$-\frac{e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}G_1(p)p^2+e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}2R_1}{a}}G_2(p)p^2}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}\;+$$

$$\frac{e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}2R_1}{a}}P_{sp2}-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}P_{sp2}+e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}2R_1}{a}}\Theta_{02}p-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}\Theta_{02}p}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}$$

$$+\frac{e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}G_1(p)p^2-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}2R_1}{a}}G_2(p)p^2}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}=\frac{P_{sp2}\left(e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_1}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_1}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}+$$

$$\frac{P_{sp2}\left(e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}+\frac{\Theta_{02}p\left(e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}2R_1}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}2R_1}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}+$$

$$\frac{\Theta_{02}p\left(e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}-\frac{G_1(p)p^2\left(e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}-e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}-$$

$$\frac{G_2(p)p^2\left(e^{\frac{\sqrt{p}\,x}{a}}\cdot e^{-\frac{\sqrt{p}2R_1}{a}}-e^{-\frac{\sqrt{p}\,x}{a}}\cdot e^{\frac{\sqrt{p}2R_1}{a}}\right)}{p^2(e^{\frac{\sqrt{p}2R_1}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}R_2}{a}})}=\frac{P_{sp2}\left(e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}+$$

$$\frac{P_{sp2}\left(e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}+\frac{\Theta_{02}p\left(e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}+$$

$$\frac{\Theta_{02}p\left(e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}-\frac{G_1(p)p^2\left(e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}-$$

$$\frac{G_2(p)p^2\left(e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}\right)}{p^2(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}})}.$$

Then $\Theta_2(x,p)=\frac{P_{sp2}}{p^2}+\frac{\Theta_{02}p}{p^2}+\frac{P_{sp2}}{p^2}\cdot\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}+$

$$\frac{P_{sp2}}{p^2}\cdot\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}+\frac{\Theta_{02}p}{p^2}\cdot\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}+$$

$$\frac{\Theta_{02}p}{p^2}\cdot\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}-\frac{G_1(p)p^2}{p^2}\cdot\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}-$$

$$\frac{G_2(p)p^2}{p^2}\cdot\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}. \qquad (3.24)$$

The equation can be presented in the following form:

$$\Theta_2(x,p)=\frac{P_{sp2}}{p^2}+\frac{\Theta_{02}}{p}+\left(\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}\right)\cdot\left(\frac{P_{sp2}}{p^2}+\frac{\Theta_{02}}{p}-G_2(p)\right)+$$

$$\left(\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}\right)\cdot\left(\frac{P_{sp2}}{p^2}+\frac{\Theta_{02}}{p}-G_1(p)\right). \qquad (3.25)$$

An original expression can be found using the rule of linearity, the second decomposition theorem, and convolution image [70].

We use the second decomposition theorem.

The essence of the second decomposition theorem reduces to the fact that original can be found as

$$\mathrm{f}(\tau)=\sum_{p_к} Res_{p_к}\left[F(p)e^{p\tau}\right]=\sum_{p_к} Res_{p_к}\left[\frac{R(p)}{Q(p)}e^{p\tau}\right]=\sum_{к=1}^{m}\frac{R(p_k)}{Q'(p_к)}e^{p_к\tau},$$

где $p_к$ – poles of function, Res$p_к$– deduction in the point $p_к$; m – poles quantity; $R(p)$ – numerator; Q(p) –denominator; $Q'(p_к)$ – denominator derivative when the poles of the function are substituted; $R(p_к)$ – numerator when the poles of the function are substituted.

Let us find the poles from the expression $e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}$

$\sqrt{p_k}=j\frac{\pi a}{(2R_1-R_2)},\ p_k=-\frac{\pi^2a^2}{(2R_1-R_2)^2}$.

The denominator derivative of the fractions $\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}$ (3.25.1) and $\dfrac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}$ (3.25.2) will have the following form:

$Q'(p)=\dfrac{(2R_1-R_2)\left(e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}+e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}\right)}{2a\sqrt{p}}$. It follows from here that

$\frac{R(p_k)}{Q'(p_k)}\cdot e^{p_k\tau}=-j\frac{\pi^2a^2}{(2R_1-R_2)^2}\cdot\left(e^{-j\frac{\pi(2R_1-x)}{(2R_1-R_2)}}-\ e^{j\frac{\pi(R_1-x)}{(2R_1-R_2)}}\right)e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}}$, for (3.25.2) and

$\frac{R(p_k)}{Q'(p_k)}\cdot e^{p_k\tau}=-j\frac{\pi^2a^2}{(2R_1-R_2)^2}\cdot\left(e^{-j\frac{\pi(R_2-x)}{(2R_1-R_2)}}-\ e^{j\frac{\pi(R_2-x)}{(2R_1-R_2)}}\right)e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}}$,for (3.25.1).

After performing the transformations, we obtain:

$$\frac{e^{-\frac{\sqrt{p}(2R_1-x)}{a}}-e^{\frac{\sqrt{p}(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}\doteqdot-2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau},\qquad(3.26)$$

$$\frac{e^{\frac{\sqrt{p}(R_2-x)}{a}}-e^{-\frac{\sqrt{p}(R_2-x)}{a}}}{e^{\frac{\sqrt{p}(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}(2R_1-R_2)}{a}}}\doteqdot 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau}.\qquad(3.27)$$

Using the linearity rules and properties of the convolution image, we find the original expression (3.24).

The convolution image for the third term of equation (3.25) will look like this:

$(fg)(\tau)=-\ \Theta_{02}2\pi\frac{a^2}{(2R_1-R_2)^2}Sin(\pi\frac{2R_1-x}{(2R_1-R_2)^2})\int_0^{\tau}e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau_1}d\tau_1-$ $P_{sp2}\ 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\int_0^{\tau}(\tau-\tau_1)e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau_1}d\tau_1+2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot$ $Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\int_0^{\tau}G_2(\tau-\tau_1)\,e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1$. After integration, we obtain: $(fg)(\tau)=\ \Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau}-1\right)Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)-P_{sp2}\frac{2}{\pi^3a^2}\Bigg((2R_1-$ $R_2)^2\left(e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}\tau}-1\right)+\pi^2a^2\tau\Bigg)Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)+2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot$

$$Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\int_0^{\tau}G_2(\tau-\tau_1)\,e^{-\frac{\pi^2a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1.\qquad(3.27.1)$$

The convolution image for the fourth term of equation (3.25) will look like this:

$$(fg)(\tau) =$$

$$\Theta_{02} 2\pi \frac{a^2}{(2R_1-R_2)^2} Sin(\pi \frac{R_2-x}{(2R_1-R_2)^2}) \int_0^\tau e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau_1} d\tau_1 + P_{sp2}\, 2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot$$

$$Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) \int_0^\tau (\tau-\tau_1) e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau_1} d\tau_1 - 2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot$$

$$Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) \int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1$$

After integration, we obtain:

$$(fg)(\tau) = -\Theta_{02} \frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) + P_{sp2} \frac{2}{\pi^3 a^2}\Bigg((2R_1-R_2)^2 \cdot$$

$$\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \pi^2 a^2 \tau\Bigg) Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) - 2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot$$

$$Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) \int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1. \quad (3.27.2)$$

The originals of the third and fourth terms in the expression (3.25) have bee found. Using the property of linearity, we obtain an expression for the temperature change on the second grain zone.

$$\Theta_2(x,\tau) = P_{sp2}\tau + \Theta_{02} + \Theta_{02} \frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) Sin\left(\pi \frac{(2R_1-x)}{(2R_1-R_2)}\right) -$$

$$P_{sp2} \frac{2}{\pi^3 a^2}\left((2R_1-R_2)^2 \cdot \left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \pi^2 a^2 \tau\right) Sin\left(\pi \frac{(2R_1-x)}{(2R_1-R_2)}\right) +$$

$$2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot Sin\left(\pi \frac{(2R_1-x)}{(2R_1-R_2)}\right) \cdot$$

$$\int_0^\tau G_2(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 - \Theta_{02} \frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) +$$

$$P_{sp2} \frac{2}{\pi^3 a^2}\left((2R_1-R_2)^2 \cdot \left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \pi^2 a^2 \tau\right) \cdot Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) -$$

$$2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) \int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 . \quad (3.28)$$

After the transformations we obtain:

$$\Theta_2(x,\tau) = \Theta_{02} + \Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right)\left(Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right) - Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\right) +$$

$$P_{sp2}\tau + P_{sp2}\frac{2}{\pi^3 a^2}\cdot\left((2R_1-R_2)^2\cdot\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \pi^2 a^2\tau\right)\cdot$$

$$\left(Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right) - Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\right) - 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\cdot$$

$$\int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 + 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot\int_0^\tau G_2(\tau-$$

$$\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 \ . \quad (3.29)$$

For the left boundary of the section ($x=2R_1$) taking into account that $Sin\left(\pi\frac{R_2-2R_1}{2R_1-R_2}\right) = Sin(-\pi) = 0$ the expression for the temperature at the $2R_1$ boundary will have the following form:

$$\Theta_2(2R_1,\tau) = \Theta_{2.1}(\tau) = \Theta_{02} + P_{sp2}\tau \ . \quad (3.30)$$

For the right boundary of the section ($x=R_2$) the expression (3.29) will have the following form:

$$\Theta_2(R_2,\tau) = \Theta_{2.2}(\tau) = \Theta_{02} + P_{sp2}\tau \ . \quad (3.31)$$

We now turn to the solution of the heat conduction problem for the third grain section (3.10). After performing the Laplace transformation on the variable τ, the problem takes the following form.

$$\begin{cases} a^2\theta_{3xx}(x,p) - p\,\theta_3(x,p) + \frac{1}{p}P_{\mathrm{sp3}} + \Theta_{02} = 0, R_2 < x < R_3, 0 < \tau < +\infty, \\ \theta_3(R_2,p) = G_2(p),\ \theta_3(R_3,\mathrm{p}) = \mathrm{h}\left[\theta_3(R_3,\mathrm{p}) - \mathrm{T_{ig}}\,(\mathrm{p})\right],\ 0 < \tau < +\infty, \end{cases} \quad (3.32)$$

Using the boundary conditions, we solve the differential equation. Representing $\theta_3(R_3,p)$ as $G_3(p)$ we get the following solution:

$$\Theta_3(x,p) = \frac{P_{\mathrm{sp3}} + \Theta_{03}p}{p^2} -$$

$$\frac{e^{-\frac{\sqrt{p}x}{a}}(P_{\text{sp3}}e^{\frac{\sqrt{p}R_2}{a}}+\Theta_{03}pe^{\frac{\sqrt{p}R_2}{a}}-G_3(p)p^2e^{\frac{\sqrt{p}R_2}{a}}-P_{\text{sp3}}e^{\frac{\sqrt{p}R_3}{a}}-\Theta_{03}pe^{\frac{\sqrt{p}R_3}{a}}+G_2(p)p^2e^{\frac{\sqrt{p}R_3}{a}}}{p^2\left(e^{\frac{\sqrt{p}R_2}{a}}\cdot e^{-\frac{\sqrt{p}R_3}{a}}-e^{-\frac{\sqrt{p}R_2}{a}}\cdot e^{\frac{\sqrt{p}R_3}{a}}\right)}+$$

$$\frac{e^{\frac{\sqrt{p}x}{a}}(P_{\text{sp3}}e^{-\frac{\sqrt{p}R_2}{a}}+\Theta_{03}pe^{-\frac{\sqrt{p}R_2}{a}}-G_3(p)p^2e^{-\frac{\sqrt{p}R_2}{a}}-P_{\text{sp3}}e^{-\frac{\sqrt{p}R_3}{a}}-\Theta_{03}pe^{-\frac{\sqrt{p}R_3}{a}}+G_2(p)p^2e^{-\frac{\sqrt{p}R_3}{a}})}{p^2\left(e^{\frac{\sqrt{p}R_2}{a}}\cdot e^{-\frac{\sqrt{p}R_3}{a}}-e^{-\frac{\sqrt{p}R_2}{a}}\cdot e^{\frac{\sqrt{p}R_3}{a}}\right)}. \qquad (3.33)$$

Solving equation (3.33) in the same way as equation (3.23), we obtain the following as a result.

$$\Theta_3(x,\tau)=\Theta_{03}+\Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)-Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right)+$$
$$P_{sp3}\tau+P_{\text{sp3}}\frac{2}{\pi^3a^2}\cdot\left((R_2-R_3)^2\cdot\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)+\pi^2a^2\tau\right)\cdot\left(Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)-\right.$$
$$\left.Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\right)-2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot$$
$$\int_0^\tau G_2(\tau-\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1+2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot\int_0^\tau G_3(\tau-$$
$$\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1. \qquad (3.34)$$

Given the fact that dependency $G_3(\tau)=h\left(\Theta_3(R_3,\tau)-T_{ig}(\tau)\right)$, $\Theta_3(x,\tau)$ will take the following form:

$$\Theta_3(x,\tau)=\Theta_{03}+\Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)-Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right)+$$
$$P_{sp3}\tau+P_{sp3}\frac{2}{\pi^3a^2}\cdot\left((R_2-R_3)^2\cdot\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)+\pi^2a^2\tau\right)\cdot\left(Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)-\right.$$
$$\left.Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\right)-2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot$$
$$\int_0^\tau G_2(\tau-\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1+2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot h\cdot$$
$$\left(\int_0^\tau\theta_3(R_3,\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1-\int_0^\tau T_{ig}(\tau-\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1\right). \qquad (3.35)$$

For the boundary with the second zone ($x=R_2$) the expression takes the following form:

$$\Theta_3(R_2,\tau) = \Theta_{3.1}(\tau) = \Theta_{03} + P_{sp3}\tau \quad (3.36)$$

For an external border (x=R_3) the expression will look like this:

$$\Theta_3(R_3,\tau) = \Theta_{03} + P_{sp3}\tau. \quad (3.37)$$

3.2.4 The solution of boundary value problems for grain cooling after microwave heating

After the microwave field stops functioning the grain begins to cool. The boundary value problem is represented by the system (3.11), which after the Laplace transformation with the parameter τ will have the following form

$$\begin{cases} a^2\theta_{1xx}(x,o) - \mathrm{p}\,\theta(x,\mathrm{p}) + \Theta_{01}, & 0 < x < 2R_1 \\ \Theta\,(0,\mathrm{p}) = G_1(\mathrm{p}), \Theta(2\mathrm{R}_1,\mathrm{p}) = G_1(\mathrm{p}), 0 < \tau < +\infty \end{cases} \quad (3.38)$$

Solving the differential equation taking in the account the boundary conditions, we obtain:

$$\Theta_1(\mathrm{x},\mathrm{p}) = \frac{\Theta_{01}}{\mathrm{p}} + \frac{e^{\frac{\sqrt{p}\mathrm{x}}{\mathrm{a}}}\left(\Theta_{01} - G_1(\mathrm{p})\mathrm{p} - \Theta_{01}e^{\frac{\sqrt{p}\,2R_1}{a}} + G_1(\mathrm{p})\mathrm{p}e^{\frac{\sqrt{p}\,2R_1}{a}}\right)}{p\left(e^{\frac{\sqrt{p}\,2R_1}{a}} - e^{-\frac{\sqrt{p}\,2R_1}{a}}\right)} -$$

$$\frac{e^{\frac{\sqrt{p}\mathrm{x}}{\mathrm{a}}}\left(\Theta_{01} - G_1(\mathrm{p})\mathrm{p} - \Theta_{01}e^{-\frac{\sqrt{p}\,2R_1}{a}} + G_1(\mathrm{p})\mathrm{p}e^{-\frac{\sqrt{p}\,2R_1}{a}}\right)}{p\left(e^{\frac{\sqrt{p}\,2R_1}{a}} - e^{-\frac{\sqrt{p}\,2R_1}{a}}\right)} \quad (3.39)$$

Let us set the second and third fractions to the common denominator and perform the transformations.

$$\left(e^{-\frac{\sqrt{p}x}{\mathrm{a}}}\Theta_{01} - G_1(\mathrm{p})\mathrm{p}e^{-\frac{\sqrt{p}x}{\mathrm{a}}} - \Theta_{01}e^{\frac{\sqrt{p}\,2R_1}{a}}e^{-\frac{\sqrt{p}x}{\mathrm{a}}} + G_1(\mathrm{p})\mathrm{p}e^{\frac{\sqrt{p}\,2R_1}{a}}e^{-\frac{\sqrt{p}x}{\mathrm{a}}} - e^{\frac{\sqrt{p}x}{\mathrm{a}}}\Theta_{01} + \right.$$

$$\left. G_1(\mathrm{p})pe^{\frac{\sqrt{p}x}{\mathrm{a}}} + \Theta_{01}e^{-\frac{\sqrt{p}\,2R_1}{a}}e^{\frac{\sqrt{p}x}{\mathrm{a}}} - G_1(\mathrm{p})pe^{-\frac{\sqrt{p}\,2R_1}{a}}e^{\frac{\sqrt{p}x}{\mathrm{a}}}\right) / \left(p\left(e^{\frac{\sqrt{p}\,2R_1}{a}} - e^{-\frac{\sqrt{p}\,2R_1}{a}}\right)\right) =$$

$$\left(\Theta_{01}\left(e^{-\frac{\sqrt{p}\,x}{a}} - e^{\frac{\sqrt{p}x}{\mathrm{a}}}\right) - G_1(\mathrm{p})p\left(e^{-\frac{\sqrt{p}\,x}{a}} - e^{\frac{\sqrt{p}x}{\mathrm{a}}}\right) - \right.$$

$$\left.\Theta_{01}\left(e^{\frac{\sqrt{p}\,(2R_1-x)}{a}} - e^{-\frac{\sqrt{p}\,(2R_1-x)}{a}}\right) + G_1(\mathrm{p})p\left(e^{\frac{\sqrt{p}\,(2R_1-x)}{a}} - e^{-\frac{\sqrt{p}\,(2R-x)}{a}}\right)\right) /$$

$$\left(p(e^{\frac{\sqrt{p}\,2R_1}{a}}e^{-\frac{\sqrt{p}\,2R_1}{a}})\right)$$

$$=\frac{\left(e^{-\frac{\sqrt{p}\,x}{a}}-e^{\frac{\sqrt{p}x}{a}}\right)(\Theta_{01}-G_1(\mathrm{p})p)+(e^{\frac{\sqrt{p}\,(2R_1-x)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-x)}{a}})(G_1(\mathrm{p})p-\Theta_{01})}{p\,(e^{\frac{\sqrt{p}\,2R_1}{a}}-e^{-\frac{\sqrt{p}\,2R_1}{a}})} \qquad (3.40)$$

We represent this fraction as the sum of the fractions

$$\frac{e^{-\frac{\sqrt{p}\,x}{a}}-e^{\frac{\sqrt{p}x}{a}}}{e^{\frac{\sqrt{p}\,2R_1}{a}}-e^{-\frac{\sqrt{p}\,2R_1}{a}}}\frac{\Theta_{01}-G_1(\mathrm{p})p}{p}+\frac{e^{\frac{\sqrt{p}\,(2R_1-x)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}\,2R_1}{a}}-e^{-\frac{\sqrt{p}\,2R_1}{a}}}\frac{G_1(\mathrm{p})p-\Theta_{01}}{p}= -$$

$$\frac{2Sinh\,(\sqrt{p}\frac{x}{a})}{2Sinh\,(\sqrt{p}\frac{2R_1}{a})}\frac{\Theta_{01}-G_1(\mathrm{p})p}{p}+\frac{2Sinh\,(\sqrt{p}\frac{2R_1-x}{a})}{2Sinh\,(\sqrt{p}\frac{2R_1}{a})}\frac{G_1(\mathrm{p})p-\Theta_{01}}{p} \qquad (3.40.1)$$

From the tables of originals and images [64], we find the originals of the left fractions multipliers

$$-\frac{2Sinh\,(\sqrt{p}\frac{x}{a})}{2Sinh\,(\sqrt{p}\frac{2R_1}{a})}=-\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} \qquad (3.40.2)$$

$$\frac{2Sinh\,(\sqrt{p}\frac{2R_1-x}{a})}{2Sinh\,(\sqrt{p}\frac{2R_1}{a})}=\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} \qquad (3.40.3)$$

From table values of originals [70] we find originals for the right cofactors (3.40.1)

$$\frac{\Theta_{01}}{p}-\frac{G_1(\mathrm{p})p}{p}=\Theta_{01}-G_1(\tau),\ \frac{G_1(\mathrm{p})p}{p}-\frac{\Theta_{01}}{p}=G_1(\tau)\text{ - }\Theta_{01}$$

Using the convolution invention, we find the original expression (3.40.1) by taking

$$f_1(\tau)=-\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau},\ f_2(\tau)=\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau},$$

$q_1= \Theta_{01}-G_1(\tau), q_2 = G_1(\tau)-\Theta_{01}$ then the convolution images of the terms from (3.40.1) can be found as follows.

$$(f_1q_1(\tau)+(f_2q_2))\cdot(\tau) = \int_0^\tau\left(-\Theta_{01}\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}+G_1(\tau-\right.$$

$$\left.\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}\right)d\tau_1+\int_0^\tau\left(G_1(\tau-\tau_1)\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}-\right.$$

$$\left.\Theta_{01}\frac{\pi}{2}\frac{a^2}{R_1^2}\mathrm{Sin}\,(\frac{\pi}{2}\frac{x}{R_1})e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}\right)d\tau_1 = \Theta_{01}\frac{2}{\pi}(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1)\,\mathrm{Sin}$$

$(\frac{\pi}{2}\frac{x}{R_1}) + \frac{\pi}{2}\frac{a^2}{R_1^2}\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\int_0^{\tau} G_1(\tau-\tau_1)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 + \frac{\pi}{2}\frac{a^2}{R_1^2}\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\int_0^{\tau} G_1(\tau -$

$\tau_1)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 + \Theta_{01}\frac{2}{\pi}\ (e^{-\pi^2\frac{a^2}{4R_1^2}\tau} - 1)\cdot Sin\ (\frac{\pi}{2}\frac{x}{R_1}) = \Theta_{01}\frac{4}{\pi}\Bigg(e^{-\pi^2\frac{a^2}{4R_1^2}\tau} -$

$1\Bigg)\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right) + \pi\frac{a^2}{R_1^2}\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\int_0^{\tau} G_1(\tau-\tau_1)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1$ The complete dependence of the temperature change in the central zone of the grain will look like this.

$$\Theta_1(x,\tau) = \Theta_{01} + \Theta_{01}\frac{4}{\pi}\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau} - 1\right)\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right) + \pi\frac{a^2}{R_1^2}\text{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\int_0^{\tau} G_1(\tau - \tau_1)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1 \quad (3.41)$$

For the center of the grain (x=R_1)

$$\Theta_1(R_{1,}\tau) = \Theta_{01} + \Theta_{01}\frac{4}{\pi}\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau} - 1\right) + \pi\frac{a^2}{R_1^2}\int_0^{\tau} G_1(\tau-\tau_1)e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1. \quad (3.42)$$

For the boundary of the central part (x=$2R_1$)

$$\Theta_1(2R_{1,}\tau) = \Theta_{01}\ . \quad (3.43)$$

After the microwave field stops functioning, the change in temperature of the second section is described by the following boundary-value problem:

$$\begin{cases} a^2\theta_{2xx}(x,p) - p\ \theta_2(x,p) + \Theta_{02} = 0, & 2R_1 < x < R_2, \\ \theta_2(R_2,p) = G_2(p),\ \ \Theta(2R_1,p) = G_1(p), & 0 < \tau < +\infty \end{cases} \quad (3.44)$$

The solution of an equation with initial and boundary conditions gives the following equation:

$$\theta_2(x,p) = \frac{\Theta_{02}}{p} - \frac{e^{-\frac{\sqrt{p}x}{a}}\left(\Theta_{02}e^{\frac{\sqrt{p}2R_1}{a}} - \Theta_{02}e^{\frac{\sqrt{p}R_2}{a}} + G_1(p)pe^{\frac{\sqrt{p}R_2}{a}} - G_2(p)pe^{\frac{\sqrt{p}2R_1}{a}}\right)}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}} - e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)}$$

$$+\frac{e^{\frac{\sqrt{p}x}{a}}\left(\Theta_{02}e^{\frac{\sqrt{p}2R_1}{a}} - \Theta_{02}e^{\frac{\sqrt{p}R_2}{a}} + G_1(p)pe^{\frac{\sqrt{p}R_2}{a}} - G_2(p)pe^{\frac{\sqrt{p}2R_1}{a}}\right)}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}} - e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)}. \quad (3.45)$$

In the second and third fractions, we expand the brackets in the numerator.

$$\theta_2(x,p)=\frac{\Theta_{02}}{p}-$$

$$\frac{\Theta_{02}e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}2R_1}{a}}-\Theta_{02}e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}R_2}{a}}+G_1(p)pe^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}R_2}{a}}-G_2(p)pe^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}2R_1}{a}}}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)}+$$

$$\frac{\Theta_{02}e^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}2R_1}{a}}-\Theta_{02}e^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}R_2}{a}}+G_1(p)pe^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-G_2(p)pe^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}2R_1}{a}}}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)}.$$

Let us perform the transformations of the second and third fractions.

$$\theta_2(x,p)=\frac{\Theta_{02}}{p}+\frac{\Theta_{02}\left(e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}R_2}{a}}-e^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}R_2}{a}}\right)+\Theta_0\left(e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}x}{a}}-e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}2R_1}{a}}\right)}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)}+$$

$$\frac{G_1(p)p\left(e^{\frac{\sqrt{p}x}{a}}e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}R_2}{a}}\right)+G_2(p)p\left(e^{-\frac{\sqrt{p}x}{a}}e^{\frac{\sqrt{p}2R_1}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}e^{\frac{\sqrt{p}x}{a}}\right)}{p\left(e^{\frac{\sqrt{p}2R_1}{a}}\cdot e^{-\frac{\sqrt{p}R_2}{a}}-e^{-\frac{\sqrt{p}2R_1}{a}}\cdot e^{\frac{\sqrt{p}R_2}{a}}\right)},$$

or

$$\theta_2(x,p)=\frac{\Theta_{02}}{p}+\frac{\Theta_{02}\left(e^{\frac{\sqrt{p}\,(R_2-x)}{a}}-e^{-\frac{\sqrt{p}\,(R_2-x)}{a}}\right)+\Theta_0\left(e^{-\frac{\sqrt{p}\,(2R_1-x)}{a}}-e^{\frac{\sqrt{p}\,(2R_1-x)}{a}}\right)}{p\left(e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}\right)}+$$

$$\frac{+G_1(p)\,p\left(e^{-\frac{\sqrt{p}\,(R_2-x)}{\alpha}}-e^{\frac{\sqrt{p}\,(R_2-x)}{\alpha}}\right)+G_2(p)p\left(e^{\frac{\sqrt{p}\,(2R_1-x)}{\alpha}}-e^{-\frac{\sqrt{p}\,(2R_1-x)}{\alpha}}\right)}{p\left(e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}\right)}.$$

Representing the whole expression in the form of separate fractions, we obtain:

$$\theta_2(x,p)=\frac{\Theta_{02}}{p}+\frac{\Theta_{02}}{p}\,\frac{e^{\frac{\sqrt{p}\,(R_2-x)}{a}}-e^{-\frac{\sqrt{p}\,(R_2-x)}{a}}}{e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}}+\frac{\Theta_{02}}{p}\,\frac{e^{-\frac{\sqrt{p}\,(2R_1-x)}{a}}-e^{\frac{\sqrt{p}\,(2R_1-x)}{a}}}{e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}}+G_1(p)$$

$$\frac{(e^{-\frac{\sqrt{p}\,(R_2-x)}{\alpha}}-e^{\frac{\sqrt{p}\,(R_2-x)}{\alpha}}}{e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}}+G_2(p)\,\frac{e^{\frac{\sqrt{p}\,(2R_1-x)}{\alpha}}-e^{-\frac{\sqrt{p}\,(2R_1-x)}{\alpha}}}{e^{\frac{\sqrt{p}\,(2R_1-R_2)}{a}}-e^{-\frac{\sqrt{p}\,(2R_1-R_2)}{a}}}. \qquad (3.46)$$

As a result, we have an easy equation for the inverse Laplace transformation.

Similarly, we use equations (3.26) and (3.27) for equation (3.25). As a result of the performed transformations, the dependence $\Theta_2(x,\tau)$ will look like this:

$$\Theta_2(x,\tau) = \Theta_{02} + \Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right) + 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot$$

$$Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\int_0^\tau G_2(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 - \Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} -\right.$$

$$\left.1\right) Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right) -$$

$$2\pi\frac{a^2}{(2R_1-R_2)^2} Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1.$$

After the transformation the equation will have the following form:

$$\Theta_2(x,\tau) = \Theta_{02} + \Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right)\left(Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right) - Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\right) -$$

$$2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\cdot\int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 + 2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot$$

$$Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot\int_0^\tau G_2(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1. \qquad (3.47)$$

For the left boundary of the section ($x = 2R_1$), the expression takes the following form $\Theta_2(2R_1,\tau) = \Theta_{02}$ (3.48)

: Accordingly, for the right boundary of the section ($x = R_2$), the expression takes the following form: $\Theta_2(R_2,\tau) = \Theta_{02}$. (3.49)

After the microwave field stops functioning, the change in the temperature of the third section is described by the following equation with the boundary conditions:

$$\begin{cases} a^2\theta_3(x,p) - p\,\theta_3(x,p) + \Theta_{03} = 0, \;\; R_2 < x < R_3, 0 < \tau < +\infty \\ \theta_3(R_2,p) = G_2(p), \theta_3(R_3,\mathrm{p}) = \mathrm{h}\left[\theta_3(R_3,\mathrm{p}) - T_{\mathrm{ig}}(p)\right] = G_3(\mathrm{p}) \end{cases}$$

Solving the equation with boundary conditions, we obtain:

$$\theta_3(x,p) = \frac{\Theta_{03}}{p} + \frac{e^{-\frac{\sqrt{p}\,x}{a}}\left(\Theta_{03}\, e^{\frac{\sqrt{p}\,\mathrm{R}_2}{a}} - \Theta_{03}\, e^{\frac{\sqrt{p}\,\mathrm{R}_3}{a}} + G_2(p)\,p\,e^{\frac{\sqrt{p}\,\mathrm{R}_3}{a}} - G_3(p)\,p\,e^{\frac{\sqrt{p}\,\mathrm{R}_2}{a}}\right)}{p\left(e^{\frac{\sqrt{p}\,\mathrm{R}_2}{a}} e^{-\frac{\sqrt{p}\,\mathrm{R}_3}{a}} - e^{\frac{\sqrt{p}\,\mathrm{R}_3}{a}} e^{-\frac{\sqrt{p}\,\mathrm{R}_2}{a}}\right)} +$$

$$\frac{-e^{\frac{\sqrt{p}\,\mathrm{x}}{a}}\left(\Theta_{03}\, e^{-\frac{\sqrt{p}\,\mathrm{R}_2}{a}} - \Theta_{03}\, e^{-\frac{\sqrt{p}\,\mathrm{R}_3}{a}} + G_2(p)\,p\,e^{-\frac{\sqrt{p}\,\mathrm{R}_3}{a}} - G_3(\mathrm{p})\,\mathrm{p}\,e^{-\frac{\sqrt{p}\,\mathrm{R}_2}{a}}\right)}{p\left(e^{\frac{\sqrt{p}\,\mathrm{R}_2}{a}} e^{-\frac{\sqrt{p}\,\mathrm{R}_3}{a}} - e^{\frac{\sqrt{p}\,\mathrm{R}_3}{a}} e^{-\frac{\sqrt{p}\,\mathrm{R}_2}{a}}\right)} \qquad (3.50)$$

Solving by analogy with (3.45) we obtain:

$$\theta_3(x,p) = \frac{\Theta_{03}}{p} + \frac{\Theta_{03}}{p}\,\frac{e^{\frac{\sqrt{p}\,(R_3-x)}{a}} - e^{-\frac{\sqrt{p}\,(R_3-x)}{a}}}{e^{\frac{\sqrt{p}\,(R_2-R_3)}{a}} - e^{-\frac{\sqrt{p}\,(R_2-R_3)}{a}}} + \frac{\Theta_0}{p}\,\frac{e^{-\frac{\sqrt{p}\,(R_2-x)}{a}} - e^{\frac{\sqrt{p}\,(R_2-x)}{a}}}{e^{\frac{\sqrt{p}\,(R_2-R_3)}{a}} - e^{-\frac{\sqrt{p}\,(R_2-R_3)}{a}}} +$$

$$G_2(p)\,\frac{e^{-\frac{\sqrt{p}\,(R_3-x)}{a}} - e^{\frac{\sqrt{p}\,(R_3-x)}{a}}}{e^{\frac{\sqrt{p}\,(R_2-R_3)}{a}} - e^{-\frac{\sqrt{p}\,(R_2-R_3)}{a}}} + G_3(p)\,\frac{e^{\frac{\sqrt{p}\,(R_2-x)}{a}} - e^{-\frac{\sqrt{p}\,(R_2-x)}{a}}}{e^{\frac{\sqrt{p}\,(R_2-R_3)}{a}} - e^{-\frac{\sqrt{p}\,(R_2-R_3)}{a}}}\,. \qquad (3.51)$$

After completing the Laplace transformation, we obtain:

$$\Theta_3(x,\tau) = \Theta_{03} + \Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\,\tau} - 1\right)\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right) - Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right) -$$

$$2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot\int_0^\tau G_2(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\,(\tau-\tau_1)}d\tau_1 + 2\pi\frac{a^2}{(R_2-R_3)^2}\cdot$$

$$Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot\int_0^\tau G_3(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\,(\tau-\tau_1)}d\tau_1. \qquad (3.52)$$

Given that $G_3(\tau) = h\left[\theta_3(R_3,\tau) - T_{ig}(\tau)\right]$ we obtain:

$$\Theta_3(x,\tau) = \Theta_{03} + \Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\,\tau} - 1\right)\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right) - Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right) -$$

$$2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot\int_0^\tau G_2(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\,(\tau-\tau_1)}d\tau_1 + 2\pi\frac{a^2}{(R_2-R_3)^2}\cdot$$

$$Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot h\cdot$$

$$\left(\int_0^\tau \Theta_3(R_3,\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1 - \int_0^\tau T_{\text{мз}}(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1\right) \qquad (3.53)$$

For the left boundary of the section ($x = R_2$) the expression takes the following form:

$$\Theta_3(R_2,\tau) = \Theta_{03}. \qquad (3.54)$$

For the right boundary of the section ($x = R_3$), we obtain:

$$\Theta_3(R_3,\tau) = \Theta_{03}. \qquad (3.55)$$

3.2.5 Adjustment of the grain temperature change equations under microwave heating taking into the account temperature change at the sections boundaries

We have obtained the equations of temperature change in the grain zones under microwave influence (3.19), (3.29), (3.35). All of them contain undisclosed integrals with dependencies describing the boundary conditions. In addition, we obtained the equations (3.21), (3.30), (3.31), (3.36), (3.37), showing the regularities of temperature change at the boundaries of the sections, on the side of each section. We will use them and finish the analytical derivation of the temperature change equations for the grain in each section.

Change in temperature at the boundary of the 1 st and 2 nd sections. For clarity, we rewrite equations (3.21) and (3.30).

$$\Theta_1(2R_1,\tau) = \Theta_{1.1}(\tau) = \Theta_{01} + P_{sp1}\tau - \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)Sin(\,\pi a) - P_{sp1}\frac{2}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)\right)\cdot Sin(\,\pi a) + \frac{\pi}{2}\frac{a^2}{R_1^2}\cdot Sin(\,\pi a)\int_0^{\tau} G_1(\tau - \tau_1)\,e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1}d\tau_1, \qquad (3.21)$$

Setting the last term to zero in (3.21), we obtain

$$\Theta_{1.1}(\tau) = \Theta_{01} + P_{sp1}\tau - \Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)Sin(\,\pi a) - P_{sp1}\frac{2}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)\right)\cdot Sin(\,\pi a),$$

$$\Theta_2(2R_1,\tau) = \Theta_{2.1}(\tau) = \Theta_{02} + P_{sp2}\tau\,. \qquad (3.30)$$

Taking into account the fact that $G_1(\tau) = \frac{\theta_{1.1}(\tau)+\theta_{2.1}(\tau)}{2}$, we obtain

$$G_1(\tau) = \frac{\Theta_{01}+P_{sp1}\tau-\Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)Sin(\,\pi a)}{2} - \frac{P_{sp1}\frac{2}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau+4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau}-1\right)\right)\cdot Sin(\,\pi a)}{2} + \frac{\Theta_{02}+P_{sp2}\tau}{2} = \frac{(\Theta_{02}+\Theta_{01})+\tau\left(P_{sp1}+P_{sp2}\right)}{2} - \Theta_{01}\frac{1}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)Sin(\,\pi a) - P_{sp1}\frac{1}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau} - 1\right)\right)\cdot Sin(\,\pi a)\,, \qquad (3.56)$$

Replacing τ for τ_1 we obtain:

$$G_1(\tau_1) = \frac{(\Theta_{02}+\Theta_{01}) + \tau_1(P_{sp1}+P_{sp2})}{2} - \Theta_{01}\frac{1}{\pi}\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1} - 1\right)Sin(\,\pi a) -$$

$$P_{sp1}\frac{1}{\pi^3 a^2}\cdot\left(a^2\pi^2\tau_1 + 4R_1^2\left(e^{-\frac{\pi^2 a^2}{4\,R_1^2}\tau_1} - 1\right)\right)\cdot Sin(\,\pi a)\,. \qquad (3.57)$$

To use an expression for $G_1(\tau_1)$ in equations (3.19), (3.20) and (3.29), it is necessary to find the following integrals: $\int_0^\tau G_1(\tau_1)e^{-\frac{\pi^2 a^2}{4R_1^2}(\tau-\tau_1)}\,d\tau_1$ and $\int_0^\tau G_1(\tau_1)e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}\,d\tau_1$.

Integrating the product $G_1(\tau_1)e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}$ we obtain:

$$\int_0^\tau G_1(\tau_1)e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}\,d\tau_1 = -\,(\Theta_{01}+\Theta_{02})\cdot\frac{(2R_1-R_2)^2}{2\pi^2 a^2}\cdot\left(e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}} - 1\right) +$$

$$\left(P_{sp1}+P_{sp2}\right)\cdot\frac{(2R_1-R_2)^2}{2\pi^4 a^4}\cdot\left((2R_1-R_2)^2 e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}} + \pi^2 a^2\tau - (2R_1-R_2)^2\right) +$$

$$\Theta_{01}Sin(\,\pi a)\frac{4(2R_1-R_2)^2\cdot\left(e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}} - e^{-\frac{\pi^2 a^2 R_1^2\tau}{4}}\right)}{\pi^2 a^2\left(4R_1^3R_2 - 4R_1^4 - R_1^2R_2^2 + 4\right)} -$$

$$\Theta_{01}Sin(\,\pi a)\frac{(2R_1-R_2)^2}{\pi^3 a^2}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) - P_{sp1}\frac{(2R_1-R_2)^2}{\pi^5 a^4}Sin(\,\pi a)\cdot\Big((2R_1 -$$

$$R_2)^2 e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}} + \pi^2 a^2\tau - (2R_1-R_2)^2\Big) + P_{sp1}\cdot\frac{16R_1^2(2R_1-R_2)^2}{\pi^5 a^4}Sin(\,\pi a)\cdot$$

$$\left(\frac{e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}} - e^{-\frac{\pi^2 a^2 R_1^2\tau}{4}}}{4R_1^3R_2 - 4R_1^4 - R_1^2R_2^2 + 4}\right) - P_{sp1}\cdot\frac{4R_1^2(2R_1-R_2)^2}{\pi^5 a^4}Sin(\,\pi a)\cdot\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right).$$

After the transformations we obtain.

$$\int_0^\tau G_1(\tau_1)e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}\,d\tau_1 =$$

$$\Theta_{01} Sin(\pi a)\frac{(2R_1-R_2)^2}{\pi^3 a^2}\left(\frac{4(2R_1-R_2)^2\cdot\left(e^{-\frac{\pi^2 a^2 \tau}{(2R_1-R_2)^2}}-e^{-\frac{\pi^2 a^2 R_1^2 \tau}{4}}\right)}{\pi^2 a^2\left(4R_1^3 R_2-4R_1^4-R_1^2 R_2^2+4\right)}-\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\right)+$$

$$P_{sp1}\frac{(2R_1-R_2)^2}{\pi^5 a^4} Sin(\pi a)\cdot\left(\frac{16R_1^2\left(e^{-\frac{\pi^2 a^2 \tau}{(2R_1-R_2)^2}}-e^{-\frac{\pi^2 a^2 R_1^2 \tau}{4}}\right)}{4R_1^3 R_2-4R_1^4-R_1^2 R_2^2+4}-(2R_1-R_2)^2 e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-\right.$$

$$\left.\pi^2 a^2 \tau+(2R_1-R_2)^2-4R_1^2\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\right)-\ (\Theta_{01}+\Theta_{02})\cdot\frac{(2R_1-R_2)^2}{2\pi^2 a^2}\cdot$$

$$\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)+\left(P_{sp1}+P_{sp2}\right)\cdot\frac{(2R_1-R_2)^2}{2\pi^4 a^4}\cdot\left((2R_1-R_2)^2 e^{-\frac{\pi^2 a^2 \tau}{(2R_1-R_2)^2}}+\pi^2 a^2 \tau-\right.$$

$$\left.(2R_1-R_2)^2\right). \qquad (3.58)$$

In turn, integrating the product $G_1(\tau_1)\mathrm{e}^{-\frac{\pi^2 a^2}{4\ R_1^2}(\tau-\tau_1)}$, we obtain:

$$\int_0^{\tau} G_1(\tau_1)\,\mathrm{e}^{-\frac{\pi^2 a^2}{4\ R_1^2}(\tau-\tau_1)}=$$

$$-\frac{2R_1^2}{\pi^2 a^2}(\Theta_{02}+\Theta_{01})\left(e^{-\frac{\pi^2 a^2}{4\ R_1^2}\tau}-1\right)+\left(P_{sp1}+P_{sp2}\right)\left(\frac{8R_1^4}{\pi^4 a^4}\left(e^{-\frac{\pi^2 a^2}{4\ R_1^2}\tau}-1\right)+\right.$$

$$\left.\frac{2R_1^2}{\pi^2 a^2}\ \tau\right)+\Theta_{01}\frac{2R_1^2}{\pi^3 a^2}\left(e^{-\frac{\pi^2 a^2}{4\ R_1^2}\tau}-1\right)Sin(\pi a)(1-2R_1^2)-P_{sp1}\frac{8R_1^4}{\pi^5 a^4}\cdot\left(e^{-\frac{\pi^2 a^2}{4\ R_1^2}\tau}-\right.$$

$$\left.1\right)(-2-\tau)\cdot Sin(\pi a). \qquad (3.59)$$

Inserting expression (3.59) into equation (3.19), we obtain a complete dependence of the temperature change in the central part of grain under microwave field. This equation will have the following form.

$$\Theta_1(x,\tau)=\Theta_{01}+P_{sp1}\tau+\Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\left(Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)-Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)\right)-$$

$$P_{sp1}\frac{2}{\pi^3a^2}\cdot\left(a^2\pi^2\tau+4R_1^2\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\right)\left(Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)+Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)\right)+$$

$$\frac{\pi}{2}\frac{a^2}{R_1^2}\left(Sin\left(\frac{\pi}{2}\frac{a}{R_1}x\right)+Sin\left(\frac{\pi}{2}\frac{x}{R_1}\right)\right)\left(-\frac{2R_1^2}{\pi^2a^2}(\Theta_{02}+\Theta_{01})\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)+\right.$$

$$\left(P_{sp1}+P_{sp2}\right)\left(\frac{8R_1^4}{\pi^4a^4}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)+\frac{2R_1^2}{\pi^2a^2}\,\tau\right)+\Theta_{01}\frac{2R_1^2}{\pi^3a^2}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\cdot$$

$$\left.Sin(\pi a)(1-2R_1^2)-P_{sp1}\frac{8R_1^4}{\pi^5a^4}\cdot\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)(-2-\tau)\cdot Sin(\pi a)\right).\quad(3.60)$$

For the center of the grain, the expression will have the following form:

$$\Theta_1(R_1,\tau)=\Theta_{01}+P_{sp1}\tau+\Theta_{01}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\left(1-Sin\left(\frac{\pi}{2}a\right)\right)-P_{\text{уд}1}\frac{2}{\pi^3a^2}\cdot$$

$$\left(a^2\pi^2\tau+4R_1^2\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\right)\left(Sin\left(\frac{\pi}{2}a\right)+1\right)+\frac{\pi}{2}\frac{a^2}{R_1^2}\left(Sin\left(\frac{\pi}{2}a\right)+1\right)\cdot$$

$$\left(-\frac{2R_1^2}{\pi^2a^2}(\Theta_{02}+\Theta_{01})\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)+\left(P_{sp1}+P_{sp2}\right)\left(\frac{8R_1^4}{\pi^4a^4}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)+\right.\right.$$

$$\left.\frac{2R_1^2}{\pi^2a^2}\,\tau\right)+\Theta_{01}\frac{2R_1^2}{\pi^3a^2}\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-1\right)\cdot Sin(\pi a)(1-2R_1^2)-P_{sp1}\frac{8R_1^4}{\pi^5a^4}\cdot\left(e^{-\frac{\pi^2a^2}{4\,R_1^2}\tau}-\right.$$

$$\left.\left.1\right)(-2-\tau)\cdot Sin(\pi a)\right).\quad(3.60.1)$$

The expression (3.58) will be used in equation (3.29). Since another undisclosed integral from $G_2(\tau_1)$ is present in equation (3.29), it is necessary to obtain a dependence for it. For this purpose we use equations (3.31) and (3.36).

$$\Theta_2(R_2,\tau)=\Theta_{2.2}(\tau)=\Theta_{02}+P_{sp2}\tau\,,\quad(3.31)$$

$$\Theta_3(R_2,\tau)=\Theta_{3.1}(\tau)=\Theta_{03}+P_{sp3}\tau\,.\quad(3.36)$$

Using the expression $G_2(\tau) = \frac{\theta_{2.2}(\tau)+\theta_{3.1}(\tau)}{2}$ we find $G_2(\tau_1) = G_2(\tau)$.

$$G_2(\tau_1) = \frac{\Theta_{02}+\Theta_{03}}{2} + \tau_1 \frac{P_{sp2}+P_{sp3}}{2}. \tag{3.61}$$

To use a expression for $G_2(\tau_1)$ in equations (3.29) and (3.35), it is necessary to find the following integrals: $\int_0^\tau G_2(\tau_1) e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1$ and $\int_0^\tau G_2(\tau_1) e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1$.

$$\int_0^\tau G_2(\tau_1) e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 = \int_0^\tau \left(\frac{\Theta_{02}+\Theta_{03}}{2} + \tau_1 \frac{P_{sp2}+P_{sp3}}{2}\right) e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}(\tau-\tau_1)} d\tau_1 = -\frac{\Theta_{02}+\Theta_{03}}{2} \cdot \frac{(2R_1-R_2)^2}{\pi^2 a^2} \cdot \left(e^{-\pi^2 \frac{a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \frac{P_{sp2}+P_{sp3}}{2} \cdot \frac{(2R_1-R_2)^2}{\pi^4 a^4} \left((2R_1-R_2)^2 e^{-\pi^2 \frac{a^2}{(2R_1-R_2)^2}\tau} + \pi^2 a^2 \tau - (2R_1-R_2)^2\right). \tag{3.62}$$

$$\int_0^\tau G_2(\tau_1) e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1 = \int_0^\tau \left(\frac{\Theta_{02}+\Theta_{03}}{2} + \tau_1 \frac{P_{sp2}+P_{sp3}}{2}\right) \cdot e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1 = -\frac{\Theta_{02}+\Theta_{03}}{2} \cdot \frac{(R_2-R_3)^2}{\pi^2 a^2} \cdot \left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} - 1\right) + \frac{P_{sp2}+P_{sp3}}{2} \cdot \frac{(R_2-R_3)^2}{\pi^4 a^4} \left((R_2-R_3)^2 e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} + \pi^2 a^2 \tau - (R_2-R_3)^2\right). \tag{3.63}$$

After inserting expressions (3.58) and (3.62) into (3.29), we obtain an equation for finding the temperature of grain at any point of the second zone:

$$\Theta_2(x,\tau) = \Theta_{02} + \Theta_{02} \frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right)\left(Sin\left(\pi \frac{(2R_1-x)}{(2R_1-R_2)}\right) - Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right)\right) + P_{sp2}\tau + P_{уд2} \frac{2}{\pi^3 a^2} \cdot \left((2R_1-R_2)^2 \cdot \left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau} - 1\right) + \pi^2 a^2 \tau\right) \cdot \left(Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) - Sin\left(\pi \frac{(2R_1-x)}{(2R_1-R_2)}\right)\right) - 2\pi \frac{a^2}{(2R_1-R_2)^2} \cdot Sin\left(\pi \frac{(R_2-x)}{(2R_1-R_2)}\right) \cdot$$

$$\left(\Theta_{01} Sin(\pi a)\frac{(2R_1-R_2)^2}{\pi^3 a^2}\left(\frac{4(2R_1-R_2)^2\cdot\left(e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}}-e^{-\frac{\pi^2 a^2 R_1^2\tau}{4}}\right)}{\pi^2 a^2\left(4R_1^3 R_2-4R_1^4-R_1^2 R_2^2+4\right)}-\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\right)+\right.$$

$$P_{sp1}\frac{(2R_1-R_2)^2}{\pi^5 a^4} Sin(\pi a)\cdot\left(\frac{16R_1^2\left(e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}}-e^{-\frac{\pi^2 a^2 R_1^2\tau}{4}}\right)}{4R_1^3 R_2-4R_1^4-R_1^2 R_2^2+4}-(2R_1-R_2)^2 e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-\right.$$

$$\left.\left.\pi^2 a^2\tau+(2R_1-R_2)^2-4R_1^2\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\right)\right)-(\Theta_{01}+\Theta_{02})\cdot\frac{(2R_1-R_2)^2}{2\pi^2 a^2}\cdot$$

$$\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)+\left(P_{sp1}+P_{sp2}\right)\cdot\frac{(2R_1-R_2)^2}{2\pi^4 a^4}\cdot\left((2R_1-R_2)^2 e^{-\frac{\pi^2 a^2\tau}{(2R_1-R_2)^2}}+\pi^2 a^2\tau-\right.$$

$$\left.\left.(2R_1-R_2)^2\right)\right)+2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot\left(-\frac{\Theta_{02}+\Theta_{03}}{2}\cdot\frac{(2R_1-R_2)^2}{\pi^2 a^2}\cdot\right.$$

$$\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)+\frac{P_{sp2}+P_{sp3}}{2}\cdot\frac{(2R_1-R_2)^2}{\pi^4 a^4}\left((2R_1-R_2)^2 e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}+\pi^2 a^2\tau-\right.$$

$$\left.\left.(2R_1-R_2)^2\right)\right). \qquad (3.64)$$

After finding the integral

$$\int_0^\tau \Theta_3(R_3,\tau_1)\, e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1=\int_0^\tau G_3(\tau_1)\, e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1=\int_0^\tau \left(\Theta_{03}+\right.$$

$$\left.\tau_1 P_{\text{уд}3}\right) e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)} d\tau_1=$$

$$P_{уд3}\cdot\left(\frac{(R_2-R_3)^2(\tau a^2\pi^2-(R_2-R_3)^2)}{\pi^4\,a^4}+\frac{\mathrm{e}^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}(R_2-R_3)^4}{\pi^4\,a^4}\right)-$$

$$\Theta_{03}\,\frac{\left(\mathrm{e}^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}-1\right)\cdot(R_2-R_3)^2}{\pi^2a^2}, \qquad (3.65)$$

let us find integral $\int_0^\tau T_{ig}(\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1$.

Temperature of the intergrain space $T_{мз}$ does not remain constant. Its change depends on the state of the grain layer in the microwave field. With a fixed dense layer, the loss of heat beyond its limits will be less than for a moving layer. With a fixed dense layer the maximum temperature value of the intergrain space will be grain surface temperature $T_{ig}(\tau_1)=\Theta_3(R_3,\tau_1)$, since the intergrain space will be mainly heated by heat exchange with the grain layer. In this case, the last term of equation (3.35) becomes 0. The dielectric permittivity of air is much less than the permittivity of grain, so the effect of the microwave field on the air of the intergrain space will be unessential. Consequently, it can be assumed that the temperature of intergrain space after some time becomes equal to the temperature of the grain surface $T_{ig}(\tau_1)=\Theta_3(R_3,\tau_1)(\tau_1-\tau_2)$. Where τ_2 – delay time of the intergrain space temperature over the surface temperature of the grain. Taking into account the account the mentioned above, we obtain

$$\int_0^\tau T_{ig}(\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1=$$

$$\int_0^\tau\left(\Theta_0+\tau_1P_{уд3}\right)(\tau_1-\tau_2)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1=\int_0^\tau\left(\Theta_0\tau_1-\Theta_0\tau_2+\tau_1^2P_{sp3}-\right.$$

$$\left.\tau_1\tau_2P_{sp3}\right)e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1.$$ After integrating this expression, we obtain:

$$\int_0^\tau T_{ig}(\tau_1)\,e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1=$$

$$\Theta_{03}\left(\frac{(R_2-R_3)^2(\tau a^2\pi^2-(R_2-R_3)^2)}{\pi^4\,a^4}+\frac{\mathrm{e}^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}(R_2-R_3)^4}{\pi^4\,a^4}\right)+$$

$$\Theta_{03}\frac{\tau_2\left(e^{-\frac{\pi^2a^2\tau}{(R_2-R_3)^2}}-1\right)(R_2-R_3)^2}{\pi^2a^2}+P_{sp3}\left(\frac{2(R_2-R_3)^6(1-e^{-\frac{\pi^2a^2\tau}{(R_2-R_3)^2}})}{\pi^6a^6}-\frac{2\tau(R_2-R_3)^4}{\pi^4a^4}+\right.$$

$$\left.\frac{\tau^2(R_2-R_3)^2}{\pi^2a^2}\right)-P_{sp3}\left(\frac{\tau_2(R_2-R_3)^2\left(\pi^2\tau a^2-(R_2-R_3)^2\right)}{\pi^4a^4}+\frac{\tau_2e^{-\frac{\pi^2a^2\tau}{(R_2-R_3)^2}}(R_2-R_3)^4}{\pi^4a^4}\right). \qquad (3.66)$$

Substituting this expression and equations (3.63), (3.65) into (3.35), we obtain an equation for finding temperature at any point of the third grain zone:

$$\Theta_3(x,\tau)=\Theta_{03}+\Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)-Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right)+$$

$$P_{sp3}\tau+P_{sp3}\frac{2}{\pi^3a^2}\cdot\left((R_2-R_3)^2\cdot\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)+\pi^2a^2\tau\right)\cdot\left(Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)-\right.$$

$$\left.Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\right)-2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot\left(-\frac{\Theta_{02}+\Theta_{03}}{2}\cdot\frac{(R_2-R_3)^2}{\pi^2a^2}\cdot\right.$$

$$\left(e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}-1\right)+\frac{P_{sp2}+P_{sp3}}{2}\cdot\frac{(R_2-R_3)^2}{\pi^4a^4}\left((R_2-R_3)^2e^{-\frac{\pi^2a^2}{(R_2-R_3)^2}\tau}+\pi^2a^2\tau-\right.$$

$$\left.\left.(R_2-R_3)^2\right)\right)+2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot h\cdot$$

$$\left(\left(P_{sp3}\cdot\left(\frac{(R_2-R_3)^2(\tau a^2\pi^2-(R_2-R_3)^2)}{\pi^4\,a^4}+\frac{e^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}(R_2-R_3)^4}{\pi^4\,a^4}\right)-\Theta_{03}\cdot\right.\right.$$

$$\left.\frac{\left(e^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}-1\right)\cdot(R_2-R_3)^2}{\pi^2a^2}\right)-$$

$$\left(\Theta_{03}\cdot\left(\frac{(R_2-R_3)^2(\tau a^2\pi^2-(R_2-R_3)^2)}{\pi^4\,a^4}+\frac{e^{-\pi^2\frac{a^2}{(R_2-R_3)^2}\tau}(R_2-R_3)^4}{\pi^4\,a^4}\right)+\Theta_{03}\cdot\right.$$

$$\frac{\tau_2\left(e^{-\frac{\pi^2 a^2 \tau}{(R_2-R_3)^2}}-1\right)(R_2-R_3)^2}{\pi^2 a^2}+P_{sp3}\cdot\left(\frac{2(R_2-R_3)^6\left(1-e^{-\frac{\pi^2 a^2 \tau}{(R_2-R_3)^2}}\right)}{\pi^6 a^6}-\frac{2\tau(R_2-R_3)^4}{\pi^4 a^4}+\right.$$

$$\left.\frac{\tau^2(R_2-R_3)^2}{\pi^2 a^2}\right)-P_{sp3}\cdot$$

$$\left(\frac{\tau_2(R_2-R_3)^2\left(\pi^2\tau a^2-(R_2-R_3)^2\right)}{\pi^4 a^4}+\right.$$

$$\left.\left.\left.\frac{\tau_2 e^{-\frac{\pi^2 a^2 \tau}{(R_2-R_3)^2}}(R_2-R_3)^4}{\pi^4 a^4}\right)\right)\right) \quad (3.67)\ .$$

3.2.6 Adjustment of the grain temperature change equations after the microwave field (cooling), taking into account the temperature change at the sections boundaries

To write down the complete equation for finding temperature at any point and at the center of the first grain section, it is necessary to find expressions for influence on the boundary with the second section. For this we use equations (3.43) and (3.48).

$$\Theta_1(2R_{1,}\tau)=\Theta_{01}\ . \quad (3.43)$$

$$\Theta_2(2R_1,\tau)=\Theta_{02}\ . \quad (3.48)$$

Since $G_1(\tau)=\frac{\theta_1(2R_1,\tau)+\theta_2(2R_1,\tau)}{2}=\frac{\Theta_{01}+\Theta_{02}}{2}$, then accepting $G_1(\tau_1)=G_1(\tau)$ let us find the integral $\int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{4R_1^2}(\tau-\tau_1)}\, d\tau_1$.

$$\int_0^\tau G_1(\tau-\tau_1)\, e^{-\frac{\pi^2 a^2}{4R_1^2}(\tau-\tau_1)} d\tau_1 = \int_0^\tau \frac{\Theta_{01}+\Theta_{02}}{2} e^{-\frac{\pi^2 a^2}{4R_1^2}(\tau-\tau_1)} d\tau_1 =$$

$$-2\Theta_{01}\frac{R_1^2}{\pi^2 a^2}\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right)-2\Theta_{02}\frac{R_1^2}{\pi^2 a^2}\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right)=-2\frac{R_1^2}{\pi^2 a^2}\cdot\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right)\cdot(\Theta_{01}+\Theta_{02}) \quad (3.68)$$

Then, taking into account (2.68), let us update the expression (3.41).

$$\Theta_1(x,\tau)=\Theta_{01}+\Theta_{01}\frac{4}{\pi}\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right)\mathrm{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)-(\Theta_{01}+\Theta_{02})\frac{2}{\pi}\mathrm{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\cdot\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right)= \Theta_{01}+(\Theta_{01}-\Theta_{02})\cdot\frac{2}{\pi}\mathrm{Sin}\left(\frac{\pi}{2}\frac{x}{R_1}\right)\cdot\left(e^{-\pi^2\frac{a^2}{4R_1^2}\tau}-1\right). \quad (3.69)$$

For the grain center ($x = R_1$) the expression takes the following form:

$$\Theta_1(R_1,\tau)=\Theta_{01}+(\Theta_{01}-\Theta_{02}) \; . \quad (3\ .70)$$

For the edge of the central grain zone ($x = 2R_1$), the equation takes the following form $\Theta_1(2R_1,\tau)=\Theta_{01}$

It is necessary to find the integrals to write down an equation for finding temperature in the second part of the grain (3.47) in an expanded form.

$$\int_0^\tau G_1(\tau_1)\, e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1 \text{ and } \int_0^\tau G_2(\tau_1)\, e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1.$$

$$\int_0^\tau G_1(\tau_1)\, e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1=\int_0^\tau\frac{\Theta_{01}+\Theta_{02}}{2}e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1=$$

$$-\Theta_{01}\frac{(2R_1-R_2)^2}{2\pi^2a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)-\Theta_{02}\frac{(2R_1-R_2)^2}{2\pi^2a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)=$$

$$-(\Theta_{01}+\Theta_{02})\frac{(2R_1-R_2)^2}{2\pi^2a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right). \quad (3.71)$$

$G_2(\tau_1)$can be found as: $G_2(\tau_1)=\frac{\theta_2(R_2,\tau)+\theta_3(R_2,\tau)}{2}=\frac{\Theta_{02}+\Theta_{03}}{2}$. Then

$$\int_0^\tau G_2(\tau_1)\, e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1=\int_0^\tau\frac{\Theta_{02}+\Theta_{03}}{2}e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}(\tau-\tau_1)}d\tau_1=$$

$$-(\Theta_{02}+\Theta_{03})\frac{(2R_1-R_2)^2}{2\pi^2a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right). \quad (3.72)$$

Inserting (3.71) and (3.72) into (3.47), we obtain the required expression:

$$\Theta_2(x,\tau) = \Theta_{02}+\Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\left(Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)-Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\right)+$$

$$2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\cdot(\Theta_{01}+\Theta_{02})\frac{(2R_1-R_2)^2}{2\pi^2 a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)-$$

$$2\pi\frac{a^2}{(2R_1-R_2)^2}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot(\Theta_{02}+\Theta_{03})\frac{(2R_1-R_2)^2}{2\pi^2 a^2}\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)=$$

$$\Theta_{02}+\Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\left(Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)-Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\right)+\frac{1}{\pi}\cdot$$

$$Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\cdot(\Theta_{01}+\Theta_{02})\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)-\frac{1}{\pi}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot$$

$$(\Theta_{02}+\Theta_{03})\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)=$$

$$\Theta_{02}+\Theta_{02}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(2R_1-R_2)^2}\tau}-1\right)\cdot\left(Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)-Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\right)+\frac{1}{\pi}\cdot$$

$$\left(e^{-\pi^2\frac{a^2}{(2R_1-R_2)^2}\tau}-1\right)\cdot\left(Sin\left(\pi\frac{(R_2-x)}{(2R_1-R_2)}\right)\cdot(\Theta_{01}+\Theta_{02})-\frac{1}{\pi}\cdot Sin\left(\pi\frac{(2R_1-x)}{(2R_1-R_2)}\right)\cdot\right.$$

$$\left.(\Theta_{02}+\Theta_{03})\right). \qquad (3.73)$$

We can perform similar actions to find the equation of temperature change for the third part of the grain.

$$\Theta_3(x,\tau) = \Theta_{03} + \ \Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau}-1\right)\cdot\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)-Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right)+$$

$$\frac{1}{\pi}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot(\Theta_{02}+\Theta_{03})\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau}-1\right)+2\pi\frac{a^2}{(R_2-R_3)^2}\cdot Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot$$

$$h\cdot\begin{pmatrix}-\Theta_{03}\frac{(R_2-R_3)^2}{2\pi^2 a^2}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau}-1\right)+\\ \int_0^\tau T_{ig}(\tau-\tau_1)\,e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}(\tau-\tau_1)}d\tau_1\end{pmatrix} \qquad (3.74)$$

If the temperature of the intergrain space $T_{мз}(\tau_1)$ is represented as a constant, then in this case the expression takes the following form:

$$\Theta_3(x,\tau) = \Theta_{03} + \Theta_{03}\frac{2}{\pi}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} - 1\right)\cdot\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right) - Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right) + \frac{1}{\pi}\cdot Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot(\Theta_{02}+\Theta_{03})\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} - 1\right) + \frac{1}{\pi}\cdot Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot h\cdot\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} - 1\right)\left(T_{ig} - \Theta_{03}\right) =$$

$$\Theta_{03} + \frac{1}{\pi}\left(e^{-\frac{\pi^2 a^2}{(R_2-R_3)^2}\tau} - 1\right)\cdot\left(2\Theta_{03}\left(Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right) - Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\right) + Sin\left(\pi\frac{(R_3-x)}{(R_2-R_3)}\right)\cdot(\Theta_{02}+\Theta_{03}) + Sin\left(\pi\frac{(R_2-x)}{(R_2-R_3)}\right)\cdot h\cdot(T_{\text{мз}} - \Theta_{03})\right). \qquad (3.75)$$

The equations obtained make it possible to model the process of heating cooling grain under the action of microwave field and after this effect is removed. The next chapter is devoted to the modeling of these processes.